

Arc 1  
Laurence D. Finston

This document is part of GNU 3DLDF, a package for three-dimensional drawing.

Copyright © 2007 The Free Software Foundation

GNU 3DLDF is free software; you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the Free Software Foundation; either version 2 of the License, or (at your option) any later version.

GNU 3DLDF is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU General Public License for more details.

You should have received a copy of the GNU General Public License along with GNU 3DLDF; if not, write to the Free Software Foundation, Inc.,  
51 Franklin St, Fifth Floor, Boston, MA 02110-1301 USA

Given a circular arc  $q$ , find the radius  $r$  and the angle  $\theta$  subtended by  $q$  at the center  $M$  of circle  $k$ .

Solution:

Let  $q$  be the arc of a circle  $k$  with unknown center  $M$  and unknown radius  $r$ . Let  $p_0$  and  $p_1$  be the end points of  $q$ . Let  $p_2$  be the point halfway between  $p_0$  and  $p_1$ . Let  $p_3$  be the intersection of  $q$  and the perpendicular to the line  $p_1p_0$  through  $p_2$ . Let  $p_4$  be the point halfway between  $p_0$  and  $p_3$ . Let  $p_5$  be the intersection of  $q$  and the perpendicular to  $p_0p_3$  through  $p_4$ . The lines  $p_3p_2$  and  $p_5p_4$  intersect at  $M$ . Let  $a$  be the distance from  $p_3$  to  $p_2$ , i.e., the magnitude of  $p_3 - p_2$ ,  $b$  the distance from  $p_0$  to  $p_2$ , and  $c$  the distance from  $p_0$  to  $p_3$ . Let  $\alpha$  be  $\angle p_3p_0p_2$  and  $\beta$  be  $\angle p_0p_3p_2$ . Then  $\alpha = \sin(a/c)$  and  $\beta = \sin(b/c)$ . Let  $\gamma$  be  $\angle p_4Mp_3$ . Since  $\angle p_3p_4M$  is a right angle,  $\gamma = \alpha$ . Since the arc  $p_0p_5$  is a quarter of the arc  $p_0p_1$ , the angle  $\theta$  subtended by  $q$  at  $M$  is  $4\gamma$ .

