GNU MPRIA

The GNU Multi-Precision Rational Interval Arithmetic Library
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This manual describes how to install and use the GNU Multi-Precision Rational Interval Arithmetic Library, release 0.7.3. Please report any errors in this manual to 'bug-mpria@gnu.org'.

More information about the GNU MPRIA Library can be found at the project homepage, http://www.gnu.org/software/mpria/.

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1 Introduction to MPRIA

1.1 Description

GNU MPRIA is intended to be a portable mathematical library written in C for rational interval arithmetic computations with arbitrary precision.

The basic principle of rational interval arithmetic consists in enclosing every number by a rational interval containing it: each number is stored as its lower and upper endpoints and these bounds are rational numbers; their absolute difference measures the precision. The purpose is on the right hand to obtain guaranteed results, thanks to interval computation, and on the left hand to compute accurate results, thanks to arbitrary precision arithmetic.

The arithmetic operations are extended for interval operands in such a way that the exact result of the operation belongs to the computed rational interval.

The GNU MPRIA library is built upon the GNU MP library for operating on rational numbers; see

> `https://gmplib.org/`.

1.2 Up-to-date Material

The latest information about the library can be found at the project homepage

> `http://www.gnu.org/software/mpria/`,

while the primary distribution point for stable releases is at

> `ftp://ftp.gnu.org/gnu/mpria/`.

Many sites around the world mirror '`ftp.gnu.org`', please use a mirror near you; for a full list, see

> `http://www.gnu.org/order/ftp.html`.

1.3 Mailing Lists

There are three public mailing lists of interest: one for release announcements, one for general questions and discussions about usage of the GNU MPRIA Library and one for bug reports. For more information, visit

> `http://lists.gnu.org/mailman/listinfo/bug-mpria/`.

The proper place for bug reports is '`bug-mpria@gnu.org`'. See Chapter 3 [Reporting Bugs], page 5, for information about reporting bugs.

1.4 How to use this Manual

Everyone should read Chapter 4 [MPRIA Basics], page 6. If you need to install the library yourself, then read Chapter 2 [Installing MPRIA], page 3. To use the library you will need to refer to Chapter 5 [Rational Interval Functions], page 12; for more advanced usage you want to peruse Chapter 6 [Low-Level Rational Interval Functions], page 17.

The rest of the manual can be used for later reference, although it is probably a good idea to glance through it.

2 Installing MPRIA

2.1 How to Install

For a generic installation of the MPRIA library, you have first to install a recent version of the GNU MP on your computer. You need a C compiler, preferably `gcc`, but any reasonable C compiler should work. And you need the standard Unix `make` command, plus some other standard Unix utility commands.

Then, in the MPRIA build directory, type the following commands.

1. '`./configure`'

   This will prepare the build and setup the options according to your system. You can give options to specify the install directories (instead of the default `/usr/local`), threading support, and so on. See the `INSTALL` file or the output of '`./configure --help`' for detailed information, in particular if you get error messages.

2. '`make`'

   This will compile MPRIA and create library files with respect to your platform and environment.

3. '`make check`'

   This will make sure MPRIA was built correctly. If you get error messages, please send a bug report to '`bug-mpria@gnu.org`'. See Chapter 3 [Reporting Bugs], page 5, for information about reporting bugs.

4. '`make install`'

   This will copy the C header file `mpria.h` to the '`include`' directory `/usr/local/include`, the library files (as the share object file `libmpria.so` on GNU/Linux computers) to the '`lib`' directory `/usr/local/lib`, possibly the file `mpria.info` to the '`info`' directory `/usr/local/share/info`, and some other documentation files into the document folder `/usr/local/share/doc/mpria` (or, if you passed the `--prefix` option to `configure`, using the prefix directory given as argument to `--prefix` instead of `/usr/local`).

2.2 Other '`make`' Targets

There are some other useful '`make`' targets:

- '`mpria.info`' or '`info`'

  Create or update an info version of the manual, in `mpria.info`; this file is already provided in the MPRIA source tarball.

- '`mpria.pdf`' or '`pdf`'

  Create a PDF version of the manual, in `mpria.pdf`; this file is already provided in the MPRIA source tarball.

- '`mpria.dvi`' or '`dvi`'

  Create a DVI version of the manual, in `mpria.dvi`.

- '`mpria.ps`' or '`ps`'

  Create a PostScript version of the manual, in `mpria.ps`.

- '`mpria.html`' or '`html`'

  Create a HTML version of the manual, in several pages in the folder `doc/mpria.html`; to obtain one single page HTML document, type '`makeinfo --html --no-split mpria.texi`' from the '`doc`' directory instead.

- ‘clean’
  Delete all object files and archive files, but not the configuration files.
- ‘distclean’
  Delete all generated files not included in the distribution.
- ‘uninstall’
  Delete all files copied by ‘make install’.

2.3 Known Build Problems

The installation procedure and the GNU MPRIA library itself have been only tested in some Unix-like environments. Because it has not been yet intensively tested, you may discover that the GNU MPRIA library suffers from all bugs of the underlying GNU MP library, plus many many more.

Please report any problem to ‘bug-mpria@gnu.org’. See Chapter 3 [Reporting Bugs], page 5, for information about reporting bugs.

2.4 Getting the Latest Version

The latest stable version of MPRIA is available from

ftp://ftp.gnu.org/gnu/mpria/.

3 Reporting Bugs

If you think you have found a bug in the MPRIA library, please investigate it and report it. Likewise, if you think you have figure out a valuable enhancement for the MPRIA library, please mature it and suggest it. This library has been made available to you: it is expected you will report the bugs that you find or you will suggest the enhancements that you wish.

For bug reports, please include enough information to reproduce the problem. Generally speaking, that means:

- The MPRIA library version, along with the involved GMP library version.
- A test case that makes it possible to reproduce the bug; do not forget to include instructions on how to run the test case.
- A description of what goes wrong; please clearly explain what is incorrect and in what way, whether or not you get a crash.
- Options given to `configure` other than specifying installation directories.
- The output from running '`./configure`', as printed to `stdout`, with any options used.
- The name of the involved compiler and its version; for `gcc`, get the version with '`gcc -v`', otherwise perhaps '``what `which cc` ``', or similar.
- Hardware and operating system names, versions and details; the output from '`uname -a`' along with the output from running '`./build-aux/config.guess`' should be sufficient.
- If the bug is related to `configure`, then attach the compressed contents of `config.log`.
- Anything else that you think would be helpful; when in doubt whether something is needed or not, include it since it is better to include too much than to leave out something important.

If your bug report is good, I will do my best to help you to get a corrected version of the library; if the bug report is poor, I will not do anything about it (aside of chiding you to send better bug reports).

Patches are welcome; if possible, please make them with '`diff -u`' and include `ChangeLog` entries. Please follow the existing coding style (even if you do not like it).

Please send your bug reports, your suggestions, your patches or your comments to:

> '`bug-mpria@gnu.org`'.

If you think something in this manual is unclear, or downright incorrect, or if the language needs to be improved, please send a note to the same address.

4 MPRIA Basics

As MPRIA is built upon GMP, it is very advisable to read the GMP Manual first.

4.1 Headers and Libraries

All declarations needed to use MPRIA are collected in the C header file `mpria.h`; it is designed to work with both C and C++ compilers. You should include this file in any program using MPRIA:

```
#include <mpria.h>
```

All programs using MPRIA must link against both `libmpria` and `libgmp` libraries. On typical Unix-like systems this can be done with '`-lmpria -lgmp`' (in that order), for example:

```
gcc -o myprogram myprogram.c -lmpria -lgmp
```

GMP and MPRIA libraries are both built using Libtool, thus an application can use that to link if desired (see Section "Integrating libtool" in *GNU Libtool*).

If GMP or MPRIA have been installed to non-standard locations then it may be necessary to use '`-I`' and '`-L`' compiler options to point to the right directories, and some sort of run-time path for shared libraries.

4.2 Nomenclature and Types

A *rational interval* is a closed connected set of rational numbers, it is represented in MPRIA by its endpoints which are GMP rational numbers. The C data type for these objects is `mpri_t`.

MPRIA functions operate on valid rational intervals, while their behaviour remains undefined with non-valid rational intervals; a valid rational interval is defined as follows[1]:

- A *valid rational interval* can have finite or infinite endpoints, but its left endpoint is not larger than its right endpoint and cannot be *+infinity* ($+1/0$) while the right endpoint cannot be *-infinity* ($-1/0$). Whenever the left and right endpoints are equal to a same rational q, the valid rational interval reduces then to the singleton interval $[q, q]$ which represents exactly the rational q; conversely, any rational q is perfectly represented by the singleton interval $[q, q]$.

MPRIA functions may return intervals that are not valid as input value; their semantic is defined as follows[2]:

- Whenever the left endpoint or the right endpoint is *NaN* ($0/0$), it indicates that an *invalid operation* has been performed and that the resulting rational interval has no mathematical meaning.
- Whenever the left endpoint is strictly greater than the right endpoint, it means that the resulting rational interval is the *empty interval*.

Some functions on rational intervals return a rational number. Among such functions, there are `mpri_get_left` and `mpri_get_right` that respectively return the left and right endpoints of a rational interval, and there is `mpri_diam_abs` that computes the width of a rational interval.

[1] The definition of a valid rational interval might be refined in future releases of MPRIA.

[2] The meaning of an invalid operation, the representation of the empty interval and their handling may evolve in future releases of MPRIA, according to the standardisation of interval arithmetic in *IEEE-1788* (see Appendix A [References], page 22).

Rational numbers (or *rationals* for short) and rational arithmetic functions are brought as is from the GMP library. The C data type for rationals is `mpq_t`, while their related functions start with the prefix `mpq_` (see Section "Rational Number Functions" in *The GNU MP Manual*).

For rational intervals, because their endpoints are numbers exactly representable that are meant to enclose a result not exactly representable, the notion of precision is essentially related to their width which is meant to be arbitrarily small. The *precision* of a rational interval designs the *integer binary logarithm* of the reciprocal of its width; as such, it expresses in bits. The corresponding C data type is `mpri_prec_t`.

When a MPRIA function implements some sort of convergent algorithm to return rational intervals, besides passing a precision parameter in bits to terminate the computation, a *surrounding mode* parameter specifies whether to place the *best convert* either at the left endpoint, at the right endpoint or arbitrarily. The C data type for these modes is `mpri_srnd_t`. Typically it concerns implementations based on the Euclidean algorithm (which are omnipresent).

Some MPRIA functions that involve heavy computations admit as last parameter an *assignment mode* which specifies whether to assign either only the left endpoint, only the right endpoint, or the two endpoints. The C data type for these modes is `mpri_asgmt_t`. Those functions are considered as low-level and are both appended with the capitalised suffix `_ASGMT` and wrapped by a macro that assigns the two endpoints.

4.3 Function Classes

There are four classes of functions in the MPRIA library:

1. Functions for intervals computation based on rational numbers: their names begin with `mpri_` and their associated type is `mpri_t`. This class gathers the standard computing assignment methods and concomitants, computing subroutines for rational interval approximations of quadratic irrational numbers, the four basic binary arithmetic operations and the classic unary operators built around them, and computing subroutines for rational interval approximations of elementary analytic mathematical functions. (See Chapter 5 [Rational Interval Functions], page 12.)
2. Low-level functions for rational interval approximations of analytic mathematical functions: their names are both prepended by `mpri_` and appended by `_ASGMT`, their associated type is `mpri_t` while their last parameter is an assignment mode of type `mpri_asgmt_t`. These low-level functions are not meant to be called directly but rather efficiently enwrapped within inline or macro functions. (See Chapter 6 [Low-Level Rational Interval Functions], page 17.)
3. Fast and convenient low-level functions that operate on signed integers and rational numbers: their names begin with `mpria_mpq_` and `mpria_mpz_`, respectively; their associated type are `mpz_t` and `mpq_t`, respectively. Implemented with great efficiency and handiness in mind, these functions are mainly inline and macro functions that are intensively used by the functions in the precedent categories; you are highly encouraged to employ them directly within time-critical or intricate subroutines. They intently complete rather than substitute their already furnished alikes in the GNU MP library, the prefix `mpria_` preventing from possible naming conflicts. (See Chapter 7 [Extra Number Functions], page 18.)
4. Miscellaneous functions. As memory management is inherited from the GNU MP library by design, this miscellanea essentially concerns functions for handling up different versions of the library. Two kinds of version handling function are distinguished: the functions that treat the version data of the library against which the application is effectively compiled, as such they act at compile time; the functions that deal with the version data of the library against which the application is dynamically linked, therefore they rather serve at run time. The formers are C preprocessor macros with names beginning with `MPRIA_VERSION_`,

the latters are C plain functions with names beginning with `mpria_libversion_`. (See Chapter 8 [General Library Functions], page 20.)

4.4 Variable Conventions

MPRIA functions expect output arguments before input arguments. This general rule, which is inherited from the GNU MP library, is based on an analogy with the assignment operator.

As a matter of fact, the analogy has been pushed further by allowing to use the same variable for both input and output in the same expression; this extension of the general rule is also inherited from the GNU MP library. For example, the square function, `mpri_sqr`, can be used as follows:

```
mpri_sqr (x, x);
```

what computes the set of squares of every rational number belonging to `x` and puts the results back in `x`.

As for MP variables, MPRIA variables must be initialised once before any assignment and may be cleared out after use. A (MP or) MPRIA variable should be initialised only once, or at least be cleared out between each initialisation. After such a variable has been initialised, it can be assigned numerous times; it will have the same allocated space during all its lifetime.

For efficiency reasons, avoid excessive initialising and clearing out: as a rule of thumb, initialise near the beginning of an application and clear out near its ending; better still, implement workspaces or garbage collections to pass and reuse these variables all along the computing process.

4.5 Precision Handling and Surrounding Modes

The following six PRECision parameters are predefined with respect to the *IEEE-754* standard (see Appendix A [References], page 22), except notably for the *meaningless precision*:

- `MPRI_PREC_BITS_NIL`: meaningless precision,
- `MPRI_PREC_BITS_HALF`: half precision (binary16) or 11 bits,
- `MPRI_PREC_BITS_SINGLE`: single precision (binary32) or 24 bits,
- `MPRI_PREC_BITS_DOUBLE`: double precision (binary64) or 53 bits,
- `MPRI_PREC_BITS_QUADRUPLE`: quadruple precision (binary128) or 113 bits,
- `MPRI_PREC_BITS_OCTUPLE`: octuple precision or 237 bits.

The following three SuRrouNDing modes are supported:

- `MPRI_SRND_BCAL`: Best Convert At Left endpoint,
- `MPRI_SRND_BCAA`: Best Convert At Any endpoint,
- `MPRI_SRND_BCAR`: Best Convert At Right endpoint.

4.6 Assignment Modes

The following three ASsiGnMenT modes are supported:

- `MPRI_ASGMT_OL`: assign Only Left endpoint,
- `MPRI_ASGMT_LR`: assign Left and Right endpoints,
- `MPRI_ASGMT_OR`: assign Only Right endpoint.

4.7 Memory Management

Basically MPRIA mimics and relays to the GNU MP memory management, except notably for temporary use (see Section "Memory Management" in *The GNU MP Manual*).

The `mpq_t` type is for the implementation of the `mpri_t` type what the `mpz_t` type is for the implementation of the `mpq_t` type itself: `mpri_t` variables never reduce their allocated space, as `mpq_t` variables.

All memory is allocated, reallocated and freed by passing on to the GNU MP memory functions as grabbed from `mp_get_memory_functions` (see Section "Custom Allocation" in *The GNU MP Manual*).

While GMP uses *temporary memory on the stack* (via `alloca`), MPRIA creates, passes along and intensively reuses *workspace*s for internal computation; the various created workspaces are freed before exiting with the help of the standard C `atexit` function (see Section "Cleanups on Exit" in *The GNU C Library Reference Manual*), therefore no memory leaks should be reported by tools like `valgrind` (`http://valgrind.org/`).

> **Teething Note:** At the time of writing, this internal workspace machinery is robust but **global**, read **not yet thread safe**, and no high-level function is yet implemented to free the created workspaces, or part of them, from time to time.

4.8 Autoconf

For applications using `autoconf` and its friends, the macro `mpria_AM_PATH_MPRIA` available in the file `mpria.m4` can be employed to link with the MPRIA automatically from the `configure` script. As preliminary work, this macro checks whether MPRIA is properly installed and performs compatibility test against either a specified version of the library or a default workable version of a recent major release of the library. To use this macro simply add the following line to the `configure.ac` `autoconf` input file:

```
mpria_AM_PATH_MPRIA([MPRIA_VERSION],
                    [action-if-found],
                    [action-if-not-found])
```

where the arguments are optional. The first argument `MPRIA_VERSION` should be either the one digit version number MAJOR, the two digit dotted version number MAJOR.MINOR or the three digit dotted version number MAJOR.MINOR.MICRO of the required release of the GNU MPRIA library. While `action-if-found` might be worthily empty or `:`, a suitable choice for `action-if-not-found` is

```
AC_MSG_ERROR([no suitable GNU MPRIA library found])
```

Then the variables `MPRIA_CPPFLAGS`, `MPRIA_CFLAGS`, `MPRIA_LDFLAGS` and `MPRIA_LIBS` can be added to the `Makefile.am` `automake` input files to obtain the correct preprocessor, compiler and linker flags. For example:

```
libfoo_la_CPPFLAGS = $(MPRIA_CPPFLAGS) $(GMP_CPPFLAGS)
libfoo_la_CFLAGS = $(MPRIA_CFLAGS) $(GMP_CFLAGS)
libfoo_la_SOURCES = foo-dim.c foo-dam.c foo-dom.c
libfoo_la_LDFLAGS = $(MPRIA_LDFLAGS) $(GMP_LDFLAGS)
libfoo_la_LIBADD = $(MPRIA_LIBS) $(GMP_LIBS) $(LIBM)
```

Note that the macro `mpria_AM_PATH_MPRIA` requires the macro `mpria_AM_PATH_GMP` which is provided in the file `mpria_ax_prog_path_gmp_cc.m4`; as you have already guessed, the macro `mpria_AM_PATH_GMP` is for the GNU MP library what the macro `mpria_AM_PATH_MPRIA` is for the

GNU MPRIA library. So, in the `configure.ac` file, the macro `mpria_AM_PATH_GMP` must precede the macro `mpria_AM_PATH_MPRIA`. In the previous example, the variables `GMP_CPPFLAGS`, `GMP_CFLAGS`, `GMP_LDFLAGS` and `GMP_LIBS` are furnished by the macro `mpria_AM_PATH_GMP`; the variable `LIBM` being set up by the Libtool macro `LT_LIB_M`.

For building more closely to the GNU MP library built, further tweaks are required. The main difficulty is to grab and use at proper time the compiler information stored at GNU MP build-time in the two macros `__GMP_CC` and `__GMP_CFLAGS`, which are defined in the header file `gmp.h`. Ideally this information should be first obtained with the help of a C PreProcessor (`CPP`) in such a way that the C Compiler (`CC`) could be then set up accordingly. Unfortunately, at the time of writing, the only ready-to-use `autoconf` macro meant to set up the C preprocessor to be employed, that is to say `AC_PROG_CPP`, depends to do so on the `autoconf` macro `AC_PROG_CC`, which determines with no easy comeback the C compiler to be employed: in short, the difficulty is harder than expected. As a matter of fact, the file `mpria_ax_prog_path_gmp_cc.m4` contains a bunch of macros that allows to overcome the issue in a transparent way for the final developer: the macro `mpria_AC_PROG_GMP_CC` have to be used instead of the macro `AC_PROG_CC`. Typically the `configure.ac` file may so contain something similar to the following scrap of code:

```
dnl Setup CC and CFLAGS wrt GMP:
mpria_AC_PROG_GMP_CC

dnl Checks for libraries:
dnl  the math library:
LT_LIB_M
dnl  the GMP libray:
mpria_AM_PATH_GMP([6.1.0])
dnl  the GNU MPRIA library:
mpria_AM_PATH_MPRIA([0.7.3])
```

Besides, the usage of `mpria_AC_PROG_GMP_CC` reinforces the checks done by `mpria_AM_PATH_GMP`. To allow code readability improvement, the two latter macros have been combined into the single macro `mpria_AC_PROG_PATH_GMP_CC`. The above scrap of code can thus be rewritten as follows:

```
dnl Setup CC and CFLAGS wrt GMP:
mpria_AC_PROG_PATH_GMP_CC([6.1.0])

dnl Checks for libraries:
dnl  the math library:
LT_LIB_M
dnl  the GNU MPRIA library:
mpria_AM_PATH_MPRIA([0.7.3])
```

Last but not least, non-standard installation locations of the MPRIA and GMP libraries are handled with respect to customary use; in particular, command line options are implemented in the `configure` script to specify these locations. The macro `mpria_AM_PATH_MPRIA` affords the following command line options which accept an absolute path as compulsory argument:

- `--with-mpria-prefix=PREFIX` assumes that MPRIA is installed in the `PREFIX` directory, the default assumption being `/usr/local`;

- `--with-mpria-include=PATH` specifies that `PATH` is the MPRIA `include` directory, the default being `PREFIX/include`;

- `--with-mpria-lib=PATH` specifies that `PATH` is the MPRIA `lib` directory, the default being `PREFIX/lib`.

The macros `mpria_AC_PROG_GMP_CC`, `mpria_AM_PATH_GMP` and `mpria_AC_PROG_PATH_GMP_CC` implement command line options that have exactly the same usage but for the GMP library instead: `--with-gmp-prefix`, `--with-gmp-include` and `--with-gmp-lib`, respectively. In addition, these macros declare the environment variable `GMP_GPP` as *precious*: this advanced feature enables to specify a Generic PreProcessor command for early processing of the header file `gmp.h`.

5 Rational Interval Functions

5.1 Initialisation Functions

An `mpri_t` object must be initialised before storing the first value in it: the function `mpri_init` is used for that purpose, the function `mpri_clear` clears it out.

`void mpri_init` (*mpri_t* **x**) [Inline Function]

Initialise *x* and set it to the singleton interval $[0/1, 0/1]$. Normally, a variable should be initialised once only or at least be cleared out (using `mpri_clear`) between consecutive initialisation.

`void mpri_clear` (*mpri_t* **x**) [Inline Function]

Free the space occupied by the endpoints of *x*. Make sure to call this function for all `mpri_t` variables when you are done with them.

5.2 Assignment Functions

These functions and macros assign new values to already initialised rational intervals.

`void mpri_set` (*mpri_t* **rop**, *const mpri_t* **op**) [Inline Function]

Assign *rop* from *op*.

`void MPRI_SET_ZERO` (*mpri_t* **op**) [Macro]
`void MPRI_SET_NAN` (*mpri_t* **op**) [Macro]

Set the value of *op* to the singleton intervals $[0/1, 0/1]$ (zero) and $[0/0, 0/0]$ (NaN), respectively.

`void MPRI_SET_Q` (*mpri_t* **rop**, *const mpq_t* **op**) [Macro]

Set the value of *rop* to the singleton interval $[op, op]$.

`void mpri_set_qi_z` (*mpri_t* **rop**, *const mpz_t* **op1**, *const mpz_t* **op2**, *const mpz_t* **op3**, *mpri_prec_t* **prec**, *mpri_srnd_t* **srnd**) [Macro]
`void mpri_set_qi_q` (*mpri_t* **rop**, *const mpq_t* **op1**, *const mpq_t* **op2**, *const mpq_t* **op3**, *mpri_prec_t* **prec**, *mpri_srnd_t* **srnd**) [Inline Function]

Set the value of *rop* to the best rational interval approximation of the quadratic irrational number $(op1 + \sqrt{op2})/op3$ with a **guaranteed** precision of at least *prec* bits and with respect to the surrounding *srnd*. The result remains undefined if the radicand *op2* is negative or if the divisor *op3* is zero. While the macro `mpri_set_qi_z` is its natural high-level wrapper, the inline function `mpri_set_qi_q` belongs to one of the efficient wrappers implemented around the low-level function `mpri_set_qi_z_ASGMT`.

`void mpri_set_q` (*mpri_t* **rop**, *const mpq_t* **op**, *mpri_prec_t* **prec**, *mpri_srnd_t* **srnd**) [Inline Function]
`void mpri_set_d` (*mpri_t* **rop**, *double* **op**, *mpri_prec_t* **prec**, *mpri_srnd_t* **srnd**) [Inline Function]

Set the value of *rop* to the best rational interval approximation of the number *op* (respectively, a rational number and a `double`) with a **guaranteed** precision of at least *prec* bits and with respect to the surrounding *srnd*. Both are inline wrappers efficiently built around the low-level function `mpri_set_qi_z_ASGMT`; a rational being a degenerate quadratic irrational, a `double` an approximative rational representation of a real number.

`void mpri_set_sqrt_q` (*mpri_t* `rop`, *const mpq_t* `op`, *mpri_prec_t* `prec`, *mpri_srnd_t* `srnd`) [Inline Function]

Set the value of *rop* to the best rational interval approximation of the square root of *op*, $\sqrt{op}$, with a **guaranteed** precision of at least *prec* bits and with respect to the surrounding *srnd*. The result is undefined if the radicand *op* is negative. It is an inline function that efficiently wraps around the low-level function `mpri_set_qi_z_ASGMT`.

`void mpri_set_rsqrt_q` (*mpri_t* `rop`, *const mpq_t* `op`, *mpri_prec_t* `prec`, *mpri_srnd_t* `srnd`) [Inline Function]

Set the value of *rop* to the best rational interval approximation of the reciprocal square root of *op*, literally $\sqrt{op}/op$, with a **guaranteed** precision of at least *prec* bits and with respect to the surrounding *srnd*. The result stays undefined if the operand *op* is either negative or zero. This inline function is an efficient wrapper built around the low-level function `mpri_set_qi_z_ASGMT`.

`void mpri_swap` (*mpri_t* `rop1`, *mpri_t* `rop2`) [Inline Function]

Swap the values *rop1* and *rop2* efficiently.

5.3 Interval Conversion Functions

`void mpri_get_q` (*mpq_t* `rop`, *const mpri_t* `op`) [Inline Function]

Convert *op* to a rational number, which is its centre.[1]

`double mpri_get_d` (*const mpri_t* `op`) [Function]

Convert *op* to a `double`, this conversion is the composition of `mpri_get_q` and `mpq_get_d`.

5.4 Interval Comparison Functions

`int mpri_equal` (*const mpri_t* `op1`, *const mpri_t* `op2`) [Inline Function]

Return either `1` (read *true*) if the rational intervals *op1* and *op2* are equal or `0` (read *false*) if they are non-equal.

`int mpri_is_zero` (*const mpri_t* `op`) [Inline Function]

Return `1` (read *true*) if the rational interval *op* is the singleton interval $[0/1, 0/1]$ (zero), `0` (read *false*) otherwise.

`int mpri_is_nonzero` (*const mpri_t* `op`) [Inline Function]

Return `1` (read *true*) if the rational interval *op* does not reduce to the singleton interval $[0/1, 0/1]$ (zero), `0` (read *false*) otherwise.

`int mpri_has_zero` (*const mpri_t* `op`) [Inline Function]

Return `1` (read *true*) if zero belongs to the rational interval *op*, `0` (read *false*) otherwise.

`int mpri_hasnot_zero` (*const mpri_t* `op`) [Inline Function]

Return either `-1` if the rational interval *op* is strictly negative, or `+1` if it is strictly positive, or `0` if it contains zero

[1] An other conversion choice might be made in future releases of MPRIA; to explicitly obtain the centre of a rational interval, use `mpri_mid` instead.

5.5 Interval Basic Functions

Some MPRIA functions on rational intervals return rational results, such as the diameter or the centre of a rational interval.

`void mpri_diam_abs` (*mpq_t* `rop`, *const mpri_t* `op`) [Inline Function]

Set the value of *rop* to the absolute diameter of the rational interval *op*, that is to say, to the difference between its right endpoint and its left one.

`void mpri_diam_rel` (*mpq_t* `rop`, *const mpri_t* `op`) [Function]

Set the value of *rop* to the relative diameter of the rational interval *op*, in other words, either to the difference between its right endpoint and its left one divided by the absolute value of its centre when it is not symmetric or to *NaN* ($[0/0, 0/0]$) when it is symmetric.

`void mpri_diam` (*mpq_t* `rop`, *const mpri_t* `op`) [Inline Function]

Set the value of *rop* to the relative diameter of the rational interval *op* if it does not contains zero and to its absolute diameter otherwise.

`void mpri_mig` (*mpq_t* `rop`, *const mpri_t* `op`) [Inline Function]
`void mpri_mag` (*mpq_t* `rop`, *const mpri_t* `op`) [Inline Function]

Set the value of *rop* to the mignitude and magnitude of the rational interval *op*, respectively, that is to say, to the smallest and largest absolute value of its elements, respectively.

`void mpri_mid` (*mpq_t* `rop`, *const mpri_t* `op`) [Inline Function]

Set the value of *rop* to the value of the middle of the rational interval *op*, namely, to the half sum of its endpoints.

`mpq_t mpri_lepref` (*const mpri_t* `op`) [Macro]
`mpq_t mpri_repref` (*const mpri_t* `op`) [Macro]

Return a reference to the left and right endpoint of the rational interval *op*, respectively.

`void mpri_get_left` (*mpq_t* `rop`, *const mpri_t* `op`) [Inline Function]
`void mpri_get_right` (*mpq_t* `rop`, *const mpri_t* `op`) [Inline Function]

Set the value of *rop* to the left and right endpoint of the rational interval *op*, respectively. These functions are equivalent to calling `mpq_set` with an appropriate `mpri_lepref` or `mpri_repref`. Direct use of `mpri_lepref` or `mpri_repref` is recommended instead of these functions.

`void mpri_urandomm` (*mpq_t* `rop`, *const mpri_t* `op`, *gmp_randstate_t* `state`) [Function]

Set the value of *rop* to a rational number picked up at random in the rational interval *op* according to a uniform distribution. If the rational interval *op* is not valid, the generator returns *NaN*, namely $0/0$.

> **Teething Note:** At the time of writing, it is not clear to the author which value the generator should return when the rational interval *op* is valid but infinite: as caveat, the actual infinite endpoint is returned.

The argument *state* must be initialized by calling one of the GMP random state initialization functions (see Section “Random State Initialization” in *The GNU MP Manual*) before invoking this functions.

5.6 Interval Arithmetic Functions

`void mpri_add` (*mpri_t* `rop`, *const mpri_t* `op1`, *const mpri_t* `op2`) [Inline Function]

`void mpri_add_q` (*mpri_t* `rop`, *const mpri_t* `op1`, *const mpq_t* `op2`) [Inline Function]
Set *rop* to *op1* + *op2*.

`void mpri_sub` (*mpri_t* `rop`, *const mpri_t* `op1`, *const mpri_t* `op2`) [Inline Function]
`void mpri_sub_q` (*mpri_t* `rop`, *const mpri_t* `op1`, *const mpq_t* `op2`) [Inline Function]
`void mpri_q_sub` (*mpri_t* `rop`, *const mpq_t* `op1`, *const mpri_t* `op2`) [Inline Function]
Set *rop* to *op1* − *op2*.

`void mpri_mul` (*mpri_t* `rop`, *const mpri_t* `op1`, *const mpri_t* `op2`) [Function]
`void mpri_mul_q` (*mpri_t* `rop`, *const mpri_t* `op1`, *const mpq_t* `op2`) [Inline Function]
Set *rop* to *op1* × *op2*. Multiplication by zero, passed as singleton interval $[0/1, 0/1]$ or literally, gives the singleton interval $[0/1, 0/1]$.

`void mpri_div` (*mpri_t* `rop`, *const mpri_t* `op1`, *const mpri_t* `op2`) [Function]
`void mpri_div_q` (*mpri_t* `rop`, *const mpri_t* `op1`, *const mpq_t* `op2`) [Inline Function]
`void mpri_q_div` (*mpri_t* `rop`, *const mpq_t* `op1`, *const mpri_t* `op2`) [Inline Function]
Set *rop* to *op1*/*op2*. When the dividend *op1* reduces to the singleton interval $[0/1, 0/1]$, viz. zero, the division returns the singleton interval $[0/1, 0/1]$ as result; when the divisor *op2* contains zero, the division returns $[0/0, 0/0]$, namely *NaN*.

`void mpri_neg` (*mpri_t* `rop`, *const mpri_t* `op`) [Inline Function]
Set *rop* to −*op*.

`void mpri_abs` (*mpri_t* `rop`, *const mpri_t* `op`) [Inline Function]
Set *rop* to $|op|$, the absolute value of *op*.

`void mpri_inv` (*mpri_t* `rop`, *const mpri_t* `op`) [Inline Function]
Set *rop* to $1/op$ when the rational interval *op* does not contains zero, to $[0/0, 0/0]$ (*NaN*) otherwise.

`void mpri_sqr` (*mpri_t* `rop`, *const mpri_t* `op`) [Inline Function]
Set *rop* to op^2.

`void mpri_sqrt` (*mpri_t* `rop`, *const mpri_t* `op`, *mpri_prec_t* `prec`) [Inline Function]
Set *rop* to the best rational interval approximation of the square root of *op*, $\sqrt{op}$, with a **guaranteed** precision of at least *prec* bits. If the rational interval radicand *op* is not positive, the return interval is $[0/0, 0/0]$, namely *NaN*. This inline function implements an efficient wrapper around the low-level function `mpri_set_qi_z_ASGMT`.

`void mpri_rsqrt` (*mpri_t* `rop`, *const mpri_t* `op`, *mpri_prec_t* `prec`) [Inline Function]
Set *rop* to the best rational interval approximation of the reciprocal square root of *op*, literally $\sqrt{op}/op$, with a **guaranteed** precision of at least *prec* bits. If the rational interval operand *op* is not strictly positive, the return interval is $[0/0, 0/0]$, to wit *NaN*. This inline function efficiently implements a wrapper around the low-level function `mpri_set_qi_z_ASGMT`.

`void mpri_mul_2exp` (*mpri_t* `rop`, *const mpri_t* `op`, *unsigned long int* `exponent`) [Inline Function]
Set *rop* to $op \times 2^{exponent}$.

`void mpri_div_2exp` (*mpri_t* `rop`, *const mpri_t* `op`, *unsigned long int* `exponent`) [Inline Function]
Set *rop* to $op/2^{exponent}$.

5.7 Interval Approximation of Elementary Functions

> **Teething Note:** At the time of writing, this part of the library is clearly at a very early stage as it basically contains only *one* function: more functions may be furnished in the coming minor releases, the all set of elementary functions in the next major release.

`void mpri_atan` (*mpri_t* **`rop`**, *const mpri_t* **`op`**, *mpri_prec_t* **`prec`**) [Inline Function]

Set *rop* to the best rational interval approximation of the arc-tangent of *op*, $\arctan(op)$, with a **guaranteed** precision of at least *prec* bits. This inline function straightforwardly wraps the function `mpri_2exp_atan`.

`void mpri_2exp_atan` (*mpri_t* **`rop`**, *unsigned long int* **`exponent`**, *const mpri_t* **`op`**, *mpri_prec_t* **`prec`**) [Function]

Set *rop* to the best rational interval approximation of 2 raised to *exponent* times the arc-tangent of *op*, $2^{exponent} \times \arctan(op)$, with a **guaranteed** precision of at least *prec* bits.

6 Low-Level Rational Interval Functions

6.1 Low-Level Interval Elementary Functions

`void mpri_set_qi_z_ASGMT` (*mpri_t* `rop`, *const mpz_t* `op1`, *const mpz_t* `op2`, *const mpz_t* `op3`, *mpri_prec_t* `prec`, *mpri_srnd_t* `srnd`, *mpri_asgmt_t* `asgmt`) [Function]

Set the value of *rop* to the best rational interval approximation of the quadratic irrational number $(op1 + \sqrt{op2})/op3$ with a **guaranteed** precision of at least *prec* bits and with respect to both the surrounding *srnd* and the assignment mode *asgmt*. The result remains undefined if the radicand *op2* is negative or if the divisor *op3* is zero.

6.2 Hard-Coded Numbers

The following collections of hard-coded numbers are mainly meant to serve the previous low-level functions within enwrapping inline functions or plain functions. For illustrations on how to wrap with them, peruse the header file `mpria.h`.

`const mpz_t __mpria_z_zero` [Constant]
`const mpz_t __mpria_z_pos_one` [Constant]
`const mpz_t __mpria_z_neg_one` [Constant]
`const mpz_t __mpria_z_pos_two` [Constant]
`const mpz_t __mpria_z_neg_two` [Constant]

Collection of `mpz_t` signed integers with self-explanatory names.

`const mpq_t __mpria_q_zero` [Constant]
`const mpq_t __mpria_q_pos_one` [Constant]
`const mpq_t __mpria_q_neg_one` [Constant]
`const mpq_t __mpria_q_pos_two` [Constant]
`const mpq_t __mpria_q_neg_two` [Constant]

Collection of `mpq_t` rational numbers with self-explanatory names.

`const mpri_t __mpria_ri_zero` [Constant]
`const mpri_t __mpria_ri_pos_one` [Constant]
`const mpri_t __mpria_ri_neg_one` [Constant]

Collection of `mpri_t` rational singleton intervals with self-explanatory names.

7 Extra Number Functions

7.1 Extra Rational Number Functions

`MPRIA_MPQ_SET_ZERO` (*Q*) [Macro]
`MPRIA_MPQ_SET_POS_ONE` (*Q*) [Macro]
`MPRIA_MPQ_SET_NEG_ONE` (*Q*) [Macro]
`MPRIA_MPQ_SET_NAN` (*Q*) [Macro]
`MPRIA_MPQ_SET_POS_INF` (*Q*) [Macro]
`MPRIA_MPQ_SET_NEG_INF` (*Q*) [Macro]

Set the value of the rational number Q to `0`, `+1`, `-1`, `0/0` (*NaN*), `+1/0` (+*infinity*) and `-1/0` (-*infinity*), respectively. These utility functions are implemented as plain macros (with self-explanatory names).

`MPRIA_MPQ_IS_ZERO` (*Q*) [Macro]
`MPRIA_MPQ_IS_NONZERO` (*Q*) [Macro]
`MPRIA_MPQ_IS_POSITIVE` (*Q*) [Macro]
`MPRIA_MPQ_IS_NEGATIVE` (*Q*) [Macro]
`MPRIA_MPQ_IS_STRICTLY_POSITIVE` (*Q*) [Macro]
`MPRIA_MPQ_IS_STRICTLY_NEGATIVE` (*Q*) [Macro]

Return `1` (read *true*) if the rational number Q is either zero, nonzero, positive, negative, strictly positive or strictly negative, respectively, `0` (read *false*) otherwise. These test functions are plain macro functions (with self-explanatory names).

`int mpria_mpq_is_nan` (*const mpq_t* `op`) [Inline Function]

Return `1` (read *true*) if the rational number *op* is *Not-a-Number*, `0` (read *false*) otherwise.

NaN, the acronym for Not-a-Number, has the representation 0/0.[1]

`int mpria_mpq_is_infinite` (*const mpq_t* `op`) [Inline Function]

Return `+1` if the rational number *op* is *positive infinity*, `-1` if it is *negative infinity*, `0` otherwise.

Positive and negative infinities have the representation $+1/0$ and $-1/0$, respectively;[2] they are commonly written +*infinity* and -*infinity*, respectively.

`int mpria_mpq_is_finite` (*const mpq_t* `op`) [Inline Function]

Return `1` (read *true*) if the rational number *op* is finite, `0` (read *false*) if it is either infinite or Not-a-Number.

`int mpria_mpq_sgn` (*const mpq_t* `op`) [Inline Function]

Return `+1` if the rational *op* is strictly positive, `0` if it is zero, or `-1` if it is strictly negative. Its behaviour stays undefined if its argument is *NaN* (0/0).
While its counterpart `mpq_sgn` is implemented as a macro, this function is implemented as an inline function: it evaluates its argument only once.

`int mpria_mpq_cmpabs` (*const mpq_t* `op1`, *const mpq_t* `op2`) [Function]

Compare the absolute values of the rational numbers *op1* and *op2*. Return either a positive value if $|op1|$ is strictly greater than $|op2|$, zero if $|op1|$ is equal to $|op2|$, or a negative value if $|op1|$ is strictly smaller than $|op2|$. Its behaviour remains undefined if at least one of its arguments is either -*infinity* $(-1/0)$, +*infinity* $(+1/0)$, or *NaN* (0/0).

---

[1] At the time of writing, GMP does not support *NaN* for `mpq_t` numbers.

[2] At the time of writing, GMP does not support infinities for `mpq_t` numbers.

`void mpria_mpq_min3` (*mpq_t* `rop`, *const mpq_t* `op1`, *const mpq_t* `op2`, *const mpq_t* `op3`) [Inline Function]

Set the value of *rop* to the minimum of the triplet $\{op1, op2, op3\}$. Its behaviour is undefined if the triplet contains *-infinity* ($-1/0$), *+infinity* ($+1/0$), or *NaN* ($0/0$).

7.2 Extra Signed Integer Functions

`MPRIA_MPZ_SET_ZERO` (*Z*) [Macro]
`MPRIA_MPZ_SET_POS_ONE` (*Z*) [Macro]
`MPRIA_MPZ_SET_NEG_ONE` (*Z*) [Macro]

Set the value of the signed integer *Z* to `0`, `+1` and `-1`, respectively. These utility functions are implemented as plain macros (with self-explanatory names).

`MPRIA_MPZ_IS_ZERO` (*Z*) [Macro]
`MPRIA_MPZ_IS_NONZERO` (*Z*) [Macro]
`MPRIA_MPZ_IS_POSITIVE` (*Z*) [Macro]
`MPRIA_MPZ_IS_NEGATIVE` (*Z*) [Macro]
`MPRIA_MPZ_IS_STRICTLY_POSITIVE` (*Z*) [Macro]
`MPRIA_MPZ_IS_STRICTLY_NEGATIVE` (*Z*) [Macro]

Return `1` (read *true*) if the signed integer *Z* is either zero, nonzero, positive, negative, strictly positive or strictly negative, respectively, and `0` (read *false*) otherwise. These test functions are plain macro functions (with self-explanatory names).

`int mpria_mpz_sgn` (*const mpz_t* `op`) [Inline Function]

Return `+1` if the signed integer *op* is strictly positive, `0` if it is zero, or `-1` if it is strictly negative.

While its counterpart `mpz_sgn` is implemented as a macro, this function is implemented as an inline function: it evaluates its argument only once.

`void mpria_mpz_minabs3` (*mpz_t* `rop`, *const mpz_t* `op1`, *const mpz_t* `op2`, *const mpz_t* `op3`) [Inline Function]

Set the value of *rop* to the minimum of the triplet $\{|op1|, |op2|, |op3|\}$.

8 General Library Functions

8.1 Library Version Handling

Different releases of the GNU MPRIA library are distinguished by an **authoritative** version triplet of nonnegative integer constants defined as macro constants. Utilities are implemented to efficiently check against, to numerically pack or to stringify this triplet; packed variants of the triplet are also defined as macro constants.

`MPRIA_VERSION_MAJOR` [Macro]
`MPRIA_VERSION_MINOR` [Macro]
`MPRIA_VERSION_MICRO` [Macro]

The **authoritative** version triplet, respectively, as nonnegative integer constants: the major version number, the minor version number (or revision number), the micro version number (or major patch level).

`void mpria_libversion_get_numbers (` [Function]
int * `major`, *int* * `minor`, *int* * `micro`)

Retrieve the *major*, *minor* and *micro* version numbers of the MPRIA library against which the application is currently linked. The `NULL` pointer is accepted as argument.

`int mpria_libversion_check_numbers` (*int* `major`, *int* `minor`, *int* `micro`) [Function]

Check the compatibility of the arbitrary *major*, *minor* and *micro* version numbers with their counterpart from the MPRIA library against which the application is currently linked. The returned response is as follows:

- 0 if the two version triplets are not compatible (incompatibility);
- 1 if they are compatible and exactly the same (strict or strong compatibility);
- 2 if they are compatible but not exactly the same (weak compatibility).

This function performs no action apart from checking and responding, in particular it does not cause the application to `abort` or to show up any kind of messages (it may be enwrapped within a `if else` statement to do so).

`int mpria_libversion_check` (*void*) [Macro]

Check the compatibility of the version triplet of the MPRIA library with which an application was compiled with the version triplet of the MPRIA library against which the application is currently linked. This is a convenient wrapping macro that passes the authoritative macro version numbers to the function `mpria_libversion_check_numbers`, as such it acts similarly. The most common cause for an incompatibility or a weak compatibility is that an application was compiled against one version of the MPRIA library while it is dynamically linked against a different one, what might be due to a misconfiguration, a downgrading or an upgrading. A typical usage may look like:

```
/* Check version of libmpria */
if (!(mpria_libversion_check ()))
{
  fprintf (stderr,"version miss-compatibility\n");
  fflush (stderr);
  abort ();
}
```

`MPRIA_VERSION_EXTRA` [Macro]

The extra version string suffix, only meant for development purposes. For production releases, *alpha* and *stable* ones, it must be reset to the empty string `""`.

`MPRIA_VERSION_NUMBER_PACK` (*Major*, *Minor*, *Micro*) [Macro]
`MPRIA_VERSION_STRING_PACK` (*Major*, *Minor*, *Micro*, *StrExtra*) [Macro]

Compact, respectively stringify, the arbitrary version triplet [*Major*, *Minor*, *Micro*] into a single number, resp. into a null-terminated string to which is appended the arbitrary extra version string suffix *StrExtra*.

`MPRIA_VERSION_NUMBER` [Macro]
`MPRIA_VERSION_STRING` [Macro]

The *non-authoritative* version number, respectively string, obtained by passing the **authoritative** version triplet to `MPRIA_VERSION_NUMBER_PACK`, resp. to `MPRIA_VERSION_STRING_PACK` with `MPRIA_VERSION_EXTRA` as fourth argument.

`int mpria_libversion_get_number` (*void*) [Function]
`const char * mpria_libversion_get_string` (*void*) [Function]

Retrieve the *non-authoritative* version number and string, respectively, of the MPRIA library against which the application is currently linked.

`const char * mpria_libversion` [Macro]
`const char * mpria_version` [Macro]

The version string of the MPRIA library against which the application is currently linked. While `mpria_libversion` is a convenient macro that wraps `mpria_libversion_get_string`, `mpria_version` is defined as synonymous of `mpria_libversion` with respect to the GNU MP naming scheme.

8.2 Miscellaneous Utilities

`MPRIA_STRINGIFY` (*Token*) [Macro]

Stringify *Token*.

Appendix A References

Teething Note: This is clearly a **non-exhaustive** list (in progress) of references.

- IEEE-1788, Interval Standard Working Group:
  http://grouper.ieee.org/groups/1788/.
- IEEE-754, Standard for Binary Floating-Point Arithmetic:
  http://grouper.ieee.org/groups/754/.

Appendix B GNU General Public License

Version 3, 29 June 2007

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Appendix D Indices

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