GNU Gama 2.14

Adjustment of geodetic networks
Edition 2.14 (28 November 2020)

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1 Introduction

GNU Gama package is dedicated to adjustment of geodetic networks. It is intended for use with traditional geodetic surveyings which are still used and needed in special measurements (e.g., underground or high precision engineering measurements) where the Global Positioning System (GPS) cannot be used.

In general, surveying is the technique and science of accurately determining the terrestrial or three-dimensional spatial position of points and the distances and angles between them.[1]

Adjustment is a technical term traditionally used by geodesists and surveyors which simply means "application of the least squares method to process the over-determined system of measurements" (statistical methods other than least squares are used sometimes but are not common). In other words, we have more observations than needed and we are trying to get the best estimate for adjusted observations and/or coordinates.

Adjustment of geodetic networks means that we have a set of fixed points with given coordinates, a set of points with unknown coordinates (possibly with approximate values available) and a set of observations among them. What is typical of adjustment of special geodetic measurements is that the resulting linearized system might be singular (we can have a network with no fixed points) and we are not only interested in the values of 'adjusted parameters and observations' but also in the estimates of their covariances. This is what Gama does.

Gama was originally inspired by Fortran system Geodet/PC (1990) designed by Frantisek Charamza. The GNU Gama project started at the department of mapping and cartography, faculty of Civil Engineering, Czech Technical University in Prague (CTU) about 1998 and its name is an acronym for *geodesy and mapping.* It was presented to a wider public for the first time at FIG Working Week 2000 in Prague and then at FIG Workshop and Seminar at HUT Helsinki in 2001.

The GNU Gama home page is

`http://www.gnu.org/software/gama/`

and the project is hosted on

`http://savannah.gnu.org/git/?group=gama`

GNU Gama is released under the GNU General Public License and is based on a C++ library of geodetic classes and functions and a small C++ template matrix library `matvec`. For parsing XML documents GNU Gama calls the `expat` parser version 1.1, written by James Clark. The `expat` parser is not part of the GNU Gama project, and is simply used by GNU Gama.

Adjustment in local Cartesian coordinate systems is fully supported by a command-line program `gama-local` that adjusts geodetic (free) networks of observed distances, directions, angles, height differences, 3D vectors and observed coordinates (coordinates with given variance-covariance matrix). Adjustment in global coordinate systems is supported only partly as a `gama-g3` program.

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[1] Wikipedia, `http://en.wikipedia.org/wiki/Surveying`

1.1 Download

GNU Gama can be found in the subdirectory `/gnu/gama/` on your favourite FTP GNU mirror (`http://www.gnu.org/prep/ftp.html`) or cloned from the GIT. See our project page at savannah (`http://savannah.gnu.org/projects/gama/`) for more information.

To get an anonymous read-only access to the GIT repository for the latest GNU Gama source, issue the following command

```
git clone git://git.sv.gnu.org/gama.git
```

The collection of sample networks is available separetely. To clone the `gama-local` examples from GIT use the command

```
git clone git://git.sv.gnu.org/gama/examples.git
```

1.2 Install

GNU Gama is developed and tested under GNU/Linux. A static library `libgama.lib` and executables are build in folders `lib` and `bin`. You can compile Gama easily yourself if you download the sources from a FTP server. The preferred way is to have `expat` XML parser installed on your system, if not, GNU Gama will be build with internally stored `expat` older source codes version 1.1.

Change to the directory of Gama project and issue the following commands at the shell prompt

```
$ ./configure
$ make
```

For GNU Gama test suite run

```
$ make check
```

If the script `configure` is not available (which is the case when you download source codes from a git server), you have to generate it using auxiliary script `autogen.sh`. To compile and build all binaries. Run

```
$ ./configure [--bindir=DIR --infodir=DIR]
$ make install
```

if you want also to install the binaries. You can use configure parameters if you need to change directories where user executables and info documentation should be installed.

Typically, if you want to download (see Section 1.1 [Download], page 2) and compile sources, you will run following commands:

```
$ git clone git://git.sv.gnu.org/gama.git gama
$ cd gama
$ ./autogen.sh
$ ./configure
$ make
```

You should have `expat` XML parser and SQLite library already installed on your system. For example to be able to compile Gama on Ubuntu 10.04 you have to install following packages:

```
make doxygen git automake autoconf libexpat1-dev libsqlite3-dev
```

To compile user documentation in various formats (PDF, HTML, . . .) run the following commands

```
$ cd doc/
$ make download-gendocs.sh
$ make run-gendocs.sh
```

The documentation should be in `doc/manual` directory. To compile API documentation run

```
$ doxygen
```

in your `gama` directory. Doxygen output will be in the `doxygen` directory.

1.2.1 CMake

Alternatively you can use CMake to generate makefiles for Unix, Windows, Mac OS X, OS/2, MSVC, Cygwin, MinGW or Xcode. Configuration file `CMakeLists.txt` is available from the root distribution directory. For example to build `gama-local` binary for Linux run

```
$ mkdir build
$ cd build
$ cmake ..
$ make
```

where `build` is an arbitrary directory name for *out-of-place build.*

1.2.2 pkgsrc

Gama is available for multiple operating systems, including NetBSD, via pkgsrc as geography/gama, a multi-OS multi-CPU portable packaging system. See `https://www.pkgsrc.org/` for more information.

1.2.3 Precompiled executables for Windows

Executebles of Qt based GUI `gama-q2` and command line programs `gama-local` and `gama-g3` are available from `https://sourceforge.net/projects/gnu-gama-q2`.

1.3 Program `gama-local`

Program `gama-local` is a simple command line tool for adjustment of geodetic *free networks.* It is available for GNU Linux (the main platform on which project GNU Gama is being developed), BSD or Windows.

Program `gama-local` reads input data in XML format (Chapter 2 [XML input data format for gama-local], page 9) and prints adjustment results into ASCII text file. If output file name is not given, adjustment results in XML format are sent to the standard output device. If development files for Sqlite3 (package `libsqlite3-dev`) are installed during the build, `gama-local` also supports reading adjustment input data from an sqlite3 database.

If run without arguments `gama-local` prints a short help

```
$ ./gama-local

Adjustment of local geodetic network        version: 2.14 / GNU g++
************************************
```

```
http://www.gnu.org/software/gama/

Usage: gama-local  [--input-xml] input.xml  [options]
       gama-local  [--input-xml] input.xml  --sqlitedb sqlite.db  --configuration name
       gama-local  --sqlitedb sqlite.db  --configuration name  [options]
       gama-local  --sqlitedb sqlite.db  --readonly-configuration name  [options]

Options:

--algorithm  svd | gso | cholesky | envelope
--language   en | ca | cz | du | es | fi | fr | hu | ru | ua | zh
--encoding   utf-8 | iso-8859-2 | iso-8859-2-flat | cp-1250 | cp-1251
--angular    400 | 360
--latitude   <latitude>
--ellipsoid  <ellipsoid name>
--text       adjustment_results.txt
--html       adjustment_results.html
--xml        adjustment_results.xml
--octave     adjustment_results.m
--svg        network_configuration.svg
--cov-band   covariance matrix of adjusted parameters in XML output
             n  = -1  for full covariance matrix (implicit value)
             n >=  0  covariances are computed only for bandwidth n
--iterations maximum number of iterations allowed in the linearized
             least squares algorithm (implicit value is 5)
--version
--help
```

Program `gama-local` version is followed by information on compiler used to build the program (apart from GNU `g++` compiler, other possibilities are Clang, Intel C++ compiler and Visual C++, when build under Microsoft Windows).

Program `gama-local` can read XML input from the standard input if you put `"-"` (hyphen) after the option `--input-xml`. This option is special because it is optional (you can specify XML input file name or `"-"` without it). Elective `--input-xml` enables backward compatibility with the usage of older versions.

Adjustment results (`--text`, `--xml`) and others can be similarly redirected to standard output if instead of a file name is used `"-"` string. If no output is given, XML adjustment format is implicitly send to standard output.

Option `--algorithm` enables to select numerical method used for solution of the adjustment. Implicitly is used Singular Value Decomposition (`svd`), alternatively user can decide for block matrix algorithm GSO by Frantisek Charamza, based on Gram-Schmidt orthogonalization. In both these cases, project equations are solved directly without forming *normal equations*. Third possibility is to select Cholesky decomposition of semidefinite matrix of normal equations (`cholesky`).

Option `--language` selects language used in output protocol. For example, if run with option `--language cz`, `gama-local` prints output results in Czech languague using UTF-8 encoding. Implicit value is `en` for output in English.

Option `--encoding` enables to change inplicit UTF-8 output encoding to iso-8859-2 (latin-2), iso-8859-2-flat (latin-2 without diacritics), cp-1250 (MS-EE encoding) cp-12251 (Russian encoding).

Option `--angular` selects angular units to be used in output.

Options `--latitude` and/or `--ellipsoid` are used when observed vertical and/or zenith angles need to be transformed into the projection plane. If none of these two options is explicitly used, no corrections are added to horizontal and/or zenith angles. If only one of these options is used, then implicit value for `--latitude` is 45 degrees (50 gons) and implicit ellipsoid is WGS84. Mathematical formulas for the corrections is given in the following section.

Option `--octave` is used to output simplified adjustment results for GNU Octave (`https://www.gnu.org/software/octave/`), i.e. in an .m file. The following information is give in the output file

- general adjustment paramameters (number of unknowns, observetions etc.)
- list of fixed points' ids (may be empty)
- adjustment points; ids
- adjustment indexes of unknouwns
- indexes of constrained coordinates (subset of adjustment indexes)
- approximate and adjusted coordinates (zero if not available)
- full covariance matrix of adjusted coordinates
- sparse design matrix, rhs and weights (Ax = b, P = inv(C_ll))
- and adjustment results in matrix format

In the case of free networks system of normal equations is augmented with matrix of constrains. Adjustmment can be then computed independetly in Octave and compared with results from Gama for unknown coordinates. We suggest that for comaprision of Gama and Octave results number of itereations is set to zero (`--iterations 0`).

This Octave output is currently available only for algorithm *envelope* (Gama version 2.10), also adjustment in Octave is not supported for the special case of *one fixed point and one constrained* (where normal equation cannot be directly augmented with constraints because of different number of unknowns).

Option `--cov-band` is used to reduce the number of computed covariances (cofactors) in XML adjustment output. Implicitly full matrix is written to XML output, which could degrade time efficiency for the `envelope` algorithm for sparse matrix solution. Explicit option for full covariance matrix is `--cov-band -1`, option `--cov-band 0` means that only a diagonal of covariance matrix is written to XML output, `--cov-band 1` results in computing the main diagonal and first codiagonal etc. If higher rank is specified then available, it is reduced do maximum possible value `dim-1`.

Option `--iterations` enables to set maximum number of iterations allowed in the linearized least squares algorithm. After the adjustment `gama-local` computes differences between adjusted observations computed from residuals and from adjusted coordinates. If the positional difference is higher than 0.5mm, approximate coordinates of adjusted points are updated and the whole adjustment is repeated in a new iteration. Implicit number of iterations is 5.

1.3.1 Reductions of horizontal and zenith angles

For evaluating of reductions of horizontal and zenith angles, `gama-local` computes a helper point P_1 in the center of the network. Horizontal and zenith angles observed at point P_2 are transformed to the projection plane perpendicular to the normal z_1 of the helper point P_1. Coordinates (x_2, y_2) of point P_2 are conserved, but its normal z_2 is rotated by the central angle $2\gamma_{12}$ to be parallel with z_1.

For observations from point P_2 to point P_3 we denote the zenith angle z^m_{23} and horizontal direction σ^m_{23}. Now, transformed zenith angle z_{23} and horizontal direction σ_{23} can be expressed as

$$\cos z_{23} = \cos z^m_{23} \cos 2\gamma_{12} + \sin z^m_{23} \cos(180^\circ - \sigma^m_{23}) \sin\gamma_{12},$$

$$\sin(180^\circ - \sigma^m_{23}) \cot\sigma_{23} = -\cos(180^\circ - \sigma^m_{23}) \cos 2\gamma_{12} + \cot z^m_{23} \sin 2\gamma_{12}$$

and after arrangement

$$\cos z_{23} = \cos z^m_{23} \cos 2\gamma_{12} - \sin z^m_{23} \cos\sigma^m_{23} \sin\gamma_{12},$$

$$\cot\sigma_{23} = \cot\sigma^m_{23} \cos 2\gamma_{12} + \frac{\cot z^m_{23} \sin 2\gamma_{12}}{\sin\sigma^m_{23}}$$

These formulas can be simplified for small networks, roughly up to the size of 6 kilometers, where

$$\cos 2\gamma_{12} \approx 1 \qquad \text{and} \qquad \sin 2\gamma_{12} \approx \frac{2\gamma''_{12}}{\varrho''}.$$

and

$$\cos z_{23} = \cos z^m_{23} - \sin z^m_{23} \cos\sigma^m_{23} \frac{2\gamma''_{12}}{\varrho''},$$

$$\cot\sigma_{23} = \cot\sigma^m_{23} + \frac{1}{\sin^2\sigma^m_{23}} \cot z^m_{23} \sin\sigma^m_{23} \frac{2\gamma''_{12}}{\varrho''}.$$

Comparing these expressions with first members of Taylor series

$$f(x) \approx f(x^0) + \frac{\mathrm{d}f(x^0)}{\mathrm{d}x}$$

of functions $\cos z_{23}$ and $\cot\sigma_{23}$ for $z_{23} = z^m_{23} + \triangle z_{23}$ and $\sigma_{23} = \sigma^m_{23} + \triangle\sigma_{23}$

$$\cos z_{23} = \cos z^m_{23} - \sin z^m_{23} \frac{\triangle z''_{23}}{\varrho''}$$

$$\cot\sigma_{23} = \cot\sigma^m_{23} - \frac{1}{\sin^2\sigma^m_{23}} \frac{\sigma''_{23}}{\varrho''},$$

it holds that $z_{23} = \cos z^m_{23} + \triangle z''_{23}$ and $\sigma_{23} = cos\sigma^m_{23} + \triangle\sigma''_{23}$.

Equations for reductions of horizontal and zenith angles now can be expressed as

$$z_{23} = \cos z^m_{23} + 2\gamma''_{12} \cos\sigma^m_{23}$$

$$\sigma_{23} = \sigma^m_{23} - 2\gamma''_{12} \cot z^m_{23} \sin\sigma^m_{23}.$$

1.4 Reporting bugs

Undoubtedly there are numerous bugs remaining, both in the `C++` source code and in the documentation. If you find a bug in either, please send a bug report to

bug-gama@gnu.org (`mailto:bug-gama@gnu.org`)

We will try to be as quick as possible in fixing the bugs and redistributing the fixes. If you prefere, you can always write directly to Aleš Čepek (`mailto:cepek@gnu.org`).

1.5 Contributors

The following persons (in chronological order) have made contributions to GNU Gama project: Aleš Čepek, Jiří Veselý, Petr Doubrava, Jan Pytel, Chuck Ghilani, Dan Haggman, Mauri Väisänen, John Dedrum, Jim Sutherland, Zoltan Faludi, Diego Berge, Boris Pihtin, Stéphane Kaloustian, Siki Zoltan, Anton Horpynich, Claudio Fontana, Bronislav Koska, Martin Beckett, Jiří Novák, Václav Petráš, Jokin Zurutuza, (Vim Xiang), Tomáš Kubín, Greg Troxel, Kristian Evers, Oleg Goussev, Petra Millarová and Jan Holešovský.

Jiří Veselý is the author of calculation of approximate coordinates by intersections and transformations (class Acord). Václav Petráš is the author of Chapter 4 [SQL schema SQLite and gama-local], page 29. Petra Millarová is the main author of class Acord2 and other helper classes for combinatorial solution of medians of approximate coordinates.

2 XML input data format for `gama-local`

The input data format for a local geodetic network adjustment (program `gama-local`) is defined in accordance with the definition of Extended Markup Language (XML) for description of structured data. The XML definition can be found at

`http://www.w3.org/TR/REC-xml`

Input data (points, observations and other related information) are described using XML start-end pair tags `<xxx>` and `</xxx>` and empty-element tags `<xxx/>`.

The syntax of XML `gama-local` input format is described in XML schema (XSD), the file `gama-local.xsd` is a part of the `GNU gama` distribution and can formally be validated independently on the program `gama-local`, namely in unit testing we use `xmllint` validating parser, if it is installed.

For parsing the XML input data, `gama-local` uses the XML parser `Expat` copyrighted by James Clark which is described at

`http://www.jclark.com/xml/expat.html`

`Expat` is subject to the Mozilla Public License (MPL), or may alternatively be used under the GNU General Public License (GPL) instead.

In the `gama-local` XML input, distances are given in meters, angular values in centigrades and their standard deviations (rms errors) in millimeters or centigrade seconds, respectively. Alternatively angular values in `gama-local` XML input can be given in degrees and seconds (see Section 2.1 [Angular units], page 9). At the end of this chapter an example of the `gama-local` XML input data object is given.

2.1 Angular units

Horizontal angles, directions and zenith angles in `gama-local` XML adjustment input are implicitly given in gons and their standard deviations and/or variances in centicentigons. Gon, also called centesimal grade and Neugrad (German for new grad), is 1/400-th of the circumference. For example

```
<direction  from="202" to="416" val="63.9347"  stdev="10.0" />
```

The same angular value (direction) can be expressed in degrees (sexagesimal graduation) as

```
<direction  from="202" to="416" val="57-32-28.428"  stdev="3.24" />
```

In XML adjustment input degrees are coded as a single string, where degrees (57), minutes (32) and seconds (28.428) are separated by dashes (-) with optional leading sign. Spaces are not allowed inside the string. Gons and degrees may be mixed in a single XML document but one should be careful to supply the information on standard deviations and/or covariances in the proper corresponding units.

Sexagesimal seconds (ss) are commonly called arcseconds, they are related to the metric system centicentigons (cc) as

$$ss = cc/400/100/100 * 360 * 60 * 60 = cc * 0.324.$$

Internally `gama-local` works with gons but output can be transformed to degrees using the option `--angular 360`.

2.2 Prologue

XML documents begin with an XML declaration that specifies the version of XML being used (*prolog*). In the case of `gama-local` follows the root tag `<gama-local>` with XML Schema namespace defined in attribute `xmlns`:

```
<?xml version="1.0" ?>
<gama-local xmlns="http://www.gnu.org/software/gama/gama-local">
```

GNU Gama uses non-validating parser and the XML Schema Definition namespace is not used in `gama-local` but it is essential for usage in third party software that might need XML validation.

2.3 Tags `<gama-local>` and `<network>`

A pair tag `<gama-local>` contains a single pair tag `<network>` that contains the network definition. The definition of the network is composed of three sections:

- `<description>` of the network (annotation or comments),
- network `<parameters />` and
- `<points-observations>` section.

The sections `<description>` and `<parameters />` are optional, the section `<points-observations>` is mandatory. These three sections may be presented in any order and may be repeated several times (in such a case, the corresponding sections are linked together by the software).

The pair tag `<network>` has two optional attributes `axes-xy` and `angles`. These attributes are used to describe orientation of the `xy` orthogonal coordinate system axes and the orientation of the observed angles and/or directions.

- `axes-xy="ne"` orientation of axes `x` and `y`; value `ne` implies that axis `x` is oriented north and axis `y` is oriented east. Acceptable values are `ne`, `sw`, `es`, `wn` for left-handed coordinate systems and `en`, `nw`, `se`, `ws` for right-handed coordinate systems (default value is `ne`).
- `angles="right-handed"` defines counterclockwise observed angles and/or directions, value `left-handed` defines clockwise observed angles and/or directions (default value is `left-handed`).

Many geodetic systems are right handed with `x` axis oriented east, `y` axis oriented north and counterclockwise angular observations. Example of left-handed orthogonal system with different axes orientation is coordinate system *Krovak* used in the Czech Republic where the axes `x` and `y` are oriented south and west respectively.

GNU Gama can adjust any combination of coordinate and angular systems.

Example

```
<gama-local>
<network>
   <description> ... </description>
   <parameters ... />
   <points-observations> ... </points-observations>
```

```
</network>
</gama-local>
```

It is planned in future versions of the program to allow more `<network>` tags (analysis of deformations etc.) and definitions of new tags.

2.4 Network description

The description of a geodetic network is enclosed in the start-end pair tags `<description>`. Text of the description is copied into the adjustment output and serves for easier identification of results. The text is not interpreted by the program, but it may be helpful for users.

Example

```
<description>
A short description of a geodetic network ...
</description>
```

2.5 Network parameters

The network parameters may be listed with the following optional attributes of an empty-element tag `<parameters />`

- `sigma-apr = "10"` value of a priori reference standard deviation—square root of reference variance (default value 10)
- `conf-pr = "0.95"` confidence probability used in statistical tests (dafault value 0.95)
- `tol-abs = "1000"` tolerance for identification of gross absolute terms in project equations (default value 1000 mm)
- `sigma-act = "aposteriori"` actual type of reference standard deviation use in statistical tests (`aposteriori | apriori`); default value is `aposteriori`
- `algorithm = "gso"` numerical algortihm used in the adjistment (gso, svd, cholesky, envelope).
- `languade = "en"` the language to be used in adjustment output.
- `encoding = "utf-8"` adjustment output encoding.
- `angular = "400"` output results angular units (400/360).
- `latitude = "50"`
- `ellipsoid`
- `cov-band = "-1"` the bandwith of covariance matrix of the adjusted parameters in the output XML file (-1 means all covariances).

Values of the attributes must be given either in the double-quotes (`"..."`) or in the single quotes (`'...'`). There can be *white spaces* (spaces, tabs and new-line characters) between attribute names, values, and the *equal* sign.

Example

```
<parameters sigma-apr = "15"
            conf-pr   = '0.90'
            sigma-act = "apriori" />
```

2.6 Points and observations

The points and observations section is bounded by the pair tag `<points-observations>` and contains information about points, observed horizontal directions, angles, and horizontal distances, height differences, slope distances, zenith angles, observed vectors and control coordinates.

Optional attributes of the start tag `<points-observations>` allow for the definition of default values of standard deviations corresponding to observed directions, angles, and distances.

- `direction-stdev = "..."` defines the implicit value of standard deviation of observed directions (default value is not defined)
- `angle-stdev = "..."` defines the implicit value of standard deviation of observed angles (default value is not defined)
- `zenith-angle-stdev = "..."` defines the implicit value of standard deviation of observed zenith angles (default value is not defined)
- `azimuth-stdev = "..."` defines the implicit value of standard deviation of observed azimuth angles (default value is not defined)
- `distance-stdev = "..."` defines the implicit value of standard deviation of observed distances, horizontal or slope (default value is not defined)

Implicit values of standard deviations for the observed distances are calculated from the model with three constants a, b, and c according to the formula

$$a + bD^c,$$

where a is a constant part of the model and D is the observed distance in kilometres. If the constants b and/or c are not given, default values of $b = 0$ and $c = 1$ will be used.

Example

```
<points-observations direction-stdev = "10"
                     distance-stdev  = "5 3 1" >
   <!-- ... points and observation data ... -->
</points-observations>
```

2.7 Points

Points are described by the empty-element tags `<point/>` with the following attributes:

- `id = "..."` is the point identification attribute (mandatory); point identification is not limited to *numbers*; all printable characters can be used in identification.
- `x = "..."` specifies coordinate `x`

- `y = "..."` specifies coordinate `y`
- `z = "..."` specifies coordinate `z`, point height
- `fix = "..."` specifies coordinates that are fixed in adjustment; acceptable values are `xy`, `XY`, `z`, `Z`, `xyz`, `XYZ`, `xyZ` and `XYz`.
- `adj = "..."` specifies coordinates to be adjusted (unknown parameters in adjustment); acceptable values are `xy`, `XY`, `z`, `Z`, `xyz`, `XYZ`, `xyZ` and `XYz`.

With exception of the first attribute (point id), all other attributes are optional. Decimal numbers can be used as needed.

Control coordinates marked using the `fix` parameter are not changed in the adjustment. Uppercase and lowercase notation of coordinates with the `fix` parameter are interpreted the same. Corrections are applied to the unknown parameters identified by coordinates written in lowercase characters given in the `adj` parameter. When the coordinates are written using uppercase, they are interpreted as *constrained coordinates.* If coordinates are marked with both the `fix` and `adj`, the `fix` parameter will take precedence.

Constrained coordinates are used for the regularization of free networks. If the network is not free (fixed network), the *constrained* coordinates are interpreted as other unknown parameters. In classical free networks, the *constrained* points define the regularization constraint

$$\sum dx_i^2 + dy_i^2 = \min .$$

where dx and dy are adjusted coordinate corrections and the summation index i goes over all *constrained* points. In other words, the set of the *constrained* points defines the adjustment of the free network (its shape and size) with a simultaneous transformation to the approximate coordinates of selected points. Program `gama-local` allows the definition of constrained coordinates with 1D leveling networks, 2D and 3D local networks.

Example

```
<point id="1" y="644498.590" x="1054980.484" fix="xy"  />
<point id="2" y="643654.101" x="1054933.801" adj="XY" />
<point id="403" adj="xy" />
```

2.8 Set of observations

The pair tag `<obs>` groups together a set of observations which are somehow related. A typical example is a set of directions and distances observed from one stand-point. An observation section contains a set of

- horizontal directions `<direction ... />`
- horizontal distances `<distance ... />`
- horizontal angles `<angle ... />`
- slope distances `<s-distance ... />`
- zenith angles `<z-angle ... />`
- azimuths `<azimuth ... />`

The band variance-covariance matrix of directions, distances, angles or other observations listed in one `<obs>` section may be supplied using a `<cov-mat>` pair tag with attributes `dim` (dimension) and `band` (bandwidth). The band-width of the diagonal matrix is equal to 0 and a fully-populated variance-covariance matrix has a bandwidth of `dim-1`.

Observation variances and covariances (i.e. an upper-symmetric part of the band-matrix) are written row by row between `<cov-mat>` and `</cov-mat>` tags. If present, the dimension of the variance-covariance matrix must agree with the number of observations.

The following example of variance-covariance matrix with dimension 6 and bandwidth 2 (two nonzero codiagonals and three zero codiagonals)

$$\begin{pmatrix} 1.1 & 0.1 & 0.2 & 0 & 0 & 0 \\ 0.1 & 1.2 & 0.3 & 0.4 & 0 & 0 \\ 0.2 & 0.3 & 1.3 & 0.5 & 0.6 & 0 \\ 0 & 0.4 & 0.5 & 1.4 & 0.7 & 0.8 \\ 0 & 0 & 0.6 & 0.7 & 1.5 & 0.9 \\ 0 & 0 & 0 & 0.8 & 0.9 & 1.6 \end{pmatrix}$$

is coded in XML as

```
<cov-mat dim="6" band="2">
   1.1  0.1  0.2
        1.2  0.3  0.4
             1.3  0.5  0.6
                  1.4  0.7  0.8
                       1.5  0.9
                            1.6
</cov-mat>
```

If two or more sets of directions with different orientations are observed from a stand-point, they must be placed in different `<obs>` sections. The value of an orientation angle can be explicitly stated with an attribute `orientation="..."`. Normally, it is more convenient to let the program calculate approximate values of orientations needed for the adjustment. If directions are present, then the attribute `station` must be defined.

Optional attribute `from_dh="..."` enables to enter implicit height of instrument for all observations within the `<obs>` pair tag.

Observed distances are expressed in meters, their standard deviations in millimeters. Observed directions and angles are expressed in centigrades (400) and their standard deviations in centigrade seconds.

Example

```
<obs from="418">
   <direction  to=  "2" val="0.0000"   stdev="10.0" />
   <direction  to="416" val="63.9347"  stdev="10.0" />
   <direction  to="420" val="336.3190" stdev="10.0" />
   <distance   to="420" val="246.594"  stdev="5.0"  />
</obs>

<obs from="418">
   <direction  to=  "2" val="0.0000"   />
```

```
      <direction  to="416" val="63.9347"  />
      <direction  to="420" val="336.3190" />
      <distance   to="420" val="246.594"  />

      <cov-mat dim="4" band="0">
         100.00 100.00 100.00 25.00
      </cov-mat>
   </obs>
```

2.9 Directions

Directions are expressed with the following attributes in an empty-element tag `<direction />`

- `to = "..."` target point identification
- `val = "..."` observed direction; see Section 2.1 [Angular units], page 9,
- `stdev = "..."` standard deviation (optional)
- `from_dh = "..."` instrument height (optional)
- `to_dh = "..."` reflector/target height (optional)

The standard deviation is an optional attribute. However since all observations in the adjustment must have their weights defined, the standard deviation must be given either explicitly with the attribute `stdev="..."` or implicitly with `<points-observation direction-stdev="..." >` or with a variance-covariance matrix for the given observation set. A similar approach applies to all the observations (distances, angles, etc.)

All directions in the given `<obs>` tag (see Section 2.8 [Set of observations], page 13) share a common *orientation shift*, which is an implicit adjustment unknown parameter defining relation between the stand point directions and bearings

$$direction_{AB} + orientation\ shift_A = bearing_{AB}.$$

Because one `<obs>` tag defines one orientation shift for all its directions, stand point *id* must be given in the `<obs from="id">` tag, using attribute *from*, which in turn must not be used in `<direction />` tags, to avoid unintentional discrepancies.

Example

```
<direction  to=  "2" val="0.0000"  stdev="10.0" />
<direction  to="416" val="63.9347" />
```

2.10 Horizontal distances

Distances are written using an empty-element tag `<distance />` with attributes

- `from = "..."` standpoint identification
- `to = "..."` target identification
- `val = "..."` observed horizontal distance
- `stdev = "..."` standard deviation of observed horizontal distance (optional)

- `from_dh` = `"..."` instrument height (optional)
- `to_dh` = `"..."` reflector/target height (optional)

Contrary to directions, distances in an observation set (`<obs>`) do not need to share a common stand-point. An example is set of distances observed from several stand-points with a common variance-covariance matrix.

Example

```
<distance from = "2"  to = "1" val = "659.184" />
<distance to ="422" val="228.207"  stdev="5.0"  />
<distance to ="408" val="568.341" />
```

2.11 Angles

Observed angles are expressed with the following attributes of an empty-element tag `<angle />`

- `from` = `"..."` standpoint identification (optional)
- `bs` = `"..."` backsight target identification
- `fs` = `"..."` foresight target identification
- `val` = `"..."` observed angle; see Section 2.1 [Angular units], page 9,
- `stdev` = `"..."` standard deviation (optional)
- `from_dh` = `"..."` instrument height (optional)
- `bs_dh` = `"..."` backsight reflector/target height (optional)
- `fs_dh` = `"..."` foresight reflector/target height (optional)

Similar to distance observations, one observation set may group angles observed from several standpoints.

Example

```
<angle from="433" bs="422" fs="402" val="128.6548" stdev="14.1"/>
<angle from="433" bs="422" fs="402" val="128.6548" />
<angle bs="422" fs="402" val="128.6548" stdev="14.1"/>
<angle bs="422" fs="402" val="128.6548"/>
```

2.12 Slope distances

Slope distances (space distances) are written using an empty-element tag `<s-distance />` with attributes

- `from` = `"..."` standpoint identification (optional)
- `to` = `"..."` target identification
- `val` = `"..."` observed slope distance
- `stdev` = `"..."` standard deviation of observed slope distance (optional)
- `from_dh` = `"..."` instrument height (optional)
- `to_dh` = `"..."` reflector/target height (optional)

Similar to horizontal distances, one observation set may group slope distances observed from several standpoints.

Example

```
<s-distance from = "2"  to = "1" val = "658.824" />
<s-distance to ="422" val="648.618"  stdev="5.0"  />
<s-distance to ="408" val="482.578" />
```

2.13 Zenith angles

Zenith angles are written using an empty-element tag `<z-angle />` with the following attributes

- `from = "..."` standpoint identification (optional)
- `to = "..."` target identification
- `val = "..."` observed zenith angle; see Section 2.1 [Angular units], page 9,
- `stdev = "..."` standard deviation of observed zenith angle (optional)
- `from_dh = "..."` instrument height (optional)
- `to_dh = "..."` reflector/target height (optional)

Similar to horizontal distances, one observation set may group zenith angles observed from several standpoints.

Example

```
<z-angle from = "2"  to = "1" val = "79.6548" />
<z-angle to ="422" val="85.4890"  stdev="5.0"  />
<z-angle to ="408" val="95.7319" />
```

2.14 Azimuths

The azimuth is defined in GNU Gama as an observed horizontal angle measured from the North to the given target. The true north orientation is measured by gyrotheodolites, mainly in mine surveying. In Gama azimuths' angle can be measured clockwise or counterclocwise according to the angle orientation defined in `<parameters />` tag.

Azimuths are expressed with the following attributes in an empty-element tag `<azimuth />`

- `from = "..."` standpoint identification
- `to = "..."` target point identification
- `val = "..."` observed azimuth; see Section 2.1 [Angular units], page 9,
- `stdev = "..."` standard deviation (optional)
- `from_dh = "..."` instrument height (optional)
- `to_dh = "..."` reflector/target height (optional)

The standard deviation is an optional attribute. However since all observations in the adjustment must have their weights defined, the standard deviation must be given either explicitly with the attribute `stdev="..."` or implicitly with `<points-observation azimuth-stdev="..." >` or with a variance-covariance matrix for the given observation set.

Example

```
<points-observations azimuth-stdev="15.0">

<azimuth from="1"  to=  "2" val= "96.484371" />
```

2.15 Height differences

A set of observed leveling height differences is described using the start-end tag `<height-differences>` without parameters. The `<height-differences>` tag can contain a series of height differences (at least one) and can optionally be supplied with a variance-covariance matrix. Single height differences are defined with empty tags `<dh />` having the following attributes:

- `from = "..."` standpoint identification
- `to = "..."` target identification
- `val = "..."` observed leveling height difference
- `stdev = "..."` standard deviation of levellin elevation and
- `dist = "..."` distance of leveling section (in kilometers)

If the value of standard deviation is not present and length of leveling section (in kilometres) is defined, the value of standard deviation is computed from the formula

$$m_{dh} = m_0\sqrt{D_{km}}.$$

If the value of standard deviation of the height difference is defined, information on leveling section length is ignored. A third possibility is to define a common variance-covariance matrix for all elevations in the set.

Example

```
<height-differences>
  <dh from="A" to="B" val=" 25.42" dist="18.1" />
  <dh from="B" to="C" val=" 10.34" dist=" 9.4" />
  <dh from="C" to="A" val="-35.20" dist="14.2" />
  <dh from="B" to="D" val="-15.54" dist="17.6" />
  <dh from="D" to="E" val=" 21.32" dist="13.5" />
  <dh from="E" to="C" val="  4.82" dist=" 9.9" />
  <dh from="E" to="A" val="-31.02" dist="13.8" />
  <dh from="C" to="D" val="-26.11" dist="14.0" />
</height-differences>
```

2.16 Control coordinates

Control (known) coordinates are described by the start-end pair tag `<coordinates>`. A series of points with known coordinates can be defined using the `<point />` tag. The variance-covariance matrix for the entire set of points can be created with a single `<cov-mat>` tag. In the `<point />` tags, a point identification (ID) and its coordinates (x, y and z) must be listed. Although the order of the `<point />` tag attributes is irrelevant in the corresponding variance-covariance matrix, the expected order of the coordinates is x, y and z (the horizontal coordinates x, y, or the height z might be missing, but not both). The type of the points may be defined either directly within the `<coordinates>` tag or outside of it.

Example

```
<coordinates>
   <point id="1" x="100.00" y="100.00" />
   <point id="2" z="200.00" y="200.00" x="200.00" />
   <point id="3" z="300.00" />
   <cov-mat dim="6" band="5" >
       ...  <!-- covariances for 1x 1y 2x 2y 2z 3z -->
   </cov-mat>
</coordinates>
```

2.17 Coordinate differences (vectors)

Observed coordinate differences describe relative positions of station pairs (vectors). Contrary to the observed coordinates, the variance-covariance matrix of the coordinate differences always describes all three elements of the 3D vectors.

Optional attributes of empty element tag `<vec>` for describing instrument and/or target height are

- `from_dh = "..."` instrument height
- `to_dh = "..."` target height

Example

```
<vectors>
   <vec from="id1" to="id2" dx="..." dy="..." dz="..." />
   <vec from="id2" to="id3" dx="..." dy="..." dz="..." />
   ...
   <cov-mat dim="..." band="..." >
       ..
   </cov-mat>
</vectors>
```

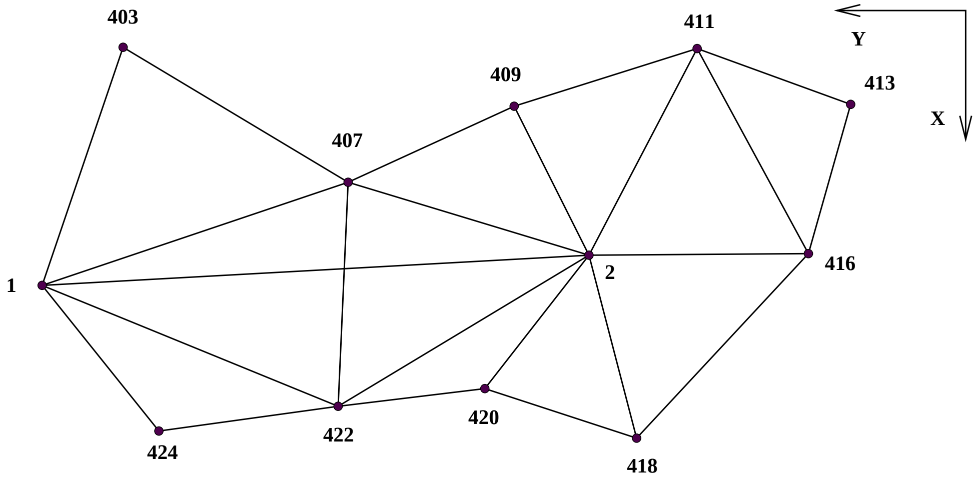
2.18 Attribute `extern`

The attribute `extern` is available for all observation types, including `<vector extern="...">` and `<coordinates extern="...">`. Its values have no impact on processing in `gama-local`, it only transferes the attribute values from XML input into the corresponing XML tags in the adjustment output.

The attribude `extern="value"` is provided to enable storing observations' database keys from an external database system in `gama-local` XML adjutement input and output. If you do not have such an external application, you probably will not need this attribute.

2.19 Example of local geodetic network

The XML input data format should be now reasonably clear from the following sample geodetic network. This example is taken from user's guide to Geodet/PC by Frantisek Charamza.



```
<?xml version="1.0" ?>

<gama-local xmlns="http://www.gnu.org/software/gama/gama-local">
<network axes-xy="sw">

<description>
XML input stream of points and observation data for the program GNU gama
</description>

<!-- parameters are expressed with empty-element tag -->

<parameters sigma-act = "aposteriori" />

<points-observations>

<!-- fixed point, constrained point -->

<point id="1" y="644498.590" x="1054980.484" fix="xy" />
<point id="2" y="643654.101" x="1054933.801" adj="XY" />
```

```
<!-- computed / adjusted points -->

<point id="403" adj="xy" />
<point id="407" adj="xy" />
<point id="409" adj="xy" />
<point id="411" adj="xy" />
<point id="413" adj="xy" />
<point id="416" adj="xy" />
<point id="418" adj="xy" />
<point id="420" adj="xy" />
<point id="422" adj="xy" />
<point id="424" adj="xy" />

<obs from="1">
     <direction  to=  "2" val=  "0.0000" stdev="10.0" />
     <direction  to="422" val= "28.2057" stdev="10.0" />
     <direction  to="424" val= "60.4906" stdev="10.0" />
     <direction  to="403" val="324.3662" stdev="10.0" />
     <direction  to="407" val="382.8182" stdev="10.0" />
     <distance   to=  "2" val= "845.777" stdev="5.0"  />
     <distance   to="422" val= "493.793" stdev="5.0"  />
     <distance   to="424" val= "288.301" stdev="5.0"  />
     <distance   to="403" val= "388.536" stdev="5.0"  />
     <distance   to="407" val= "498.750" stdev="5.0"  />
</obs>

<obs from="2">
     <direction  to=  "1" val="0.0000"   stdev="10.0" />
     <direction  to="407" val="22.2376"  stdev="10.0" />
     <direction  to="409" val="73.8984"  stdev="10.0" />
     <direction  to="411" val="134.2090" stdev="10.0" />
     <direction  to="416" val="203.0706" stdev="10.0" />
     <direction  to="418" val="287.2951" stdev="10.0" />
     <direction  to="420" val="345.6928" stdev="10.0" />
     <direction  to="422" val="368.9908" stdev="10.0" />
     <distance   to="407" val="388.562"  stdev="5.0"  />
     <distance   to="409" val="257.498"  stdev="5.0"  />
     <distance   to="411" val="360.282"  stdev="5.0"  />
     <distance   to="416" val="338.919"  stdev="5.0"  />
     <distance   to="418" val="292.094"  stdev="5.0"  />
     <distance   to="420" val="261.408"  stdev="5.0"  />
     <distance   to="422" val="452.249"  stdev="5.0"  />
</obs>

<obs from="403">
     <direction  to=  "1" val="0.0000"   stdev="10.0" />
```

```
          <direction  to="407" val="313.5542" stdev="10.0" />
          <distance   to="407" val="405.403"  stdev="5.0"  />
     </obs>

     <obs from="407">
          <direction  to=  "1" val="0.0000"   stdev="10.0" />
          <direction  to="403" val="55.1013"  stdev="10.0" />
          <direction  to="409" val="193.3410" stdev="10.0" />
          <direction  to=  "2" val="239.4204" stdev="10.0" />
          <direction  to="422" val="323.5443" stdev="10.0" />
          <distance   to="409" val="281.997"  stdev="5.0"  />
          <distance   to="422" val="346.415"  stdev="5.0"  />
     </obs>

     <obs from="409">
          <direction  to=  "2" val="0.0000"   stdev="10.0" />
          <direction  to="407" val="102.2575" stdev="10.0" />
          <direction  to="411" val="310.1751" stdev="10.0" />
          <distance   to="411" val="296.281"  stdev="5.0" />
     </obs>

     <obs from="411">
          <direction  to=  "2" val="0.0000"   stdev="10.0" />
          <direction  to="409" val="49.8647"  stdev="10.0" />
          <direction  to="413" val="291.4953" stdev="10.0" />
          <direction  to="416" val="337.6667" stdev="10.0" />
          <distance   to="413" val="252.266"  stdev="5.0"  />
          <distance   to="416" val="360.449"  stdev="5.0"  />
     </obs>

     <obs from="413">
          <direction  to="411" val="0.0000"   stdev="10.0" />
          <direction  to="416" val="295.3582" stdev="10.0" />
          <distance   to="416" val="239.745"  stdev="5.0"  />
     </obs>

     <obs from="416">
          <direction  to=  "2" val="0.0000"   stdev="10.0" />
          <direction  to="411" val="68.8065"  stdev="10.0" />
          <direction  to="413" val="117.9922" stdev="10.0" />
          <direction  to="418" val="348.1606" stdev="10.0" />
          <distance   to="418" val="389.397"  stdev="5.0"  />
     </obs>

     <obs from="418">
          <direction  to=  "2" val="0.0000"   stdev="10.0" />
          <direction  to="416" val="63.9347"  stdev="10.0" />
```

```
     <direction  to="420" val="336.3190" stdev="10.0" />
     <distance   to="420" val="246.594"  stdev="5.0"  />
</obs>

<obs from="420">
     <direction  to=  "2" val="0.0000"   stdev="10.0" />
     <direction  to="418" val="77.9221"  stdev="10.0" />
     <direction  to="422" val="250.1804" stdev="10.0" />
     <distance   to="422" val="228.207"  stdev="5.0"  />
</obs>

<obs from="422">
     <direction  to=  "2" val="0.0000"   stdev="10.0" />
     <direction  to="420" val="26.8834"  stdev="10.0" />
     <direction  to="424" val="225.7964" stdev="10.0" />
     <direction  to=  "1" val="259.2124" stdev="10.0" />
     <direction  to="407" val="337.3724" stdev="10.0" />
     <distance   to="424" val="279.405"  stdev="5.0"  />
</obs>

<obs from="424">
     <direction  to=  "1" val="0.0000"   stdev="10.0" />
     <direction  to="422" val="134.2955" stdev="10.0" />
</obs>

</points-observations>

</network>
</gama-local>
```

3 YAML input data format for `gama-local`

YAML is a human-readable data-serialization language. It is commonly used for configuration files and in applications where data is being stored or transmitted. YAML targets many of the same communications applications as Extensible Markup Language but has a minimal syntax which intentionally differs from SGML. *Wikipedia*

In version 2.12 YAML support was added for gama-local as an alternative to the existing XML input format. The YAML support is limited only to conversion program `gama-local-yaml2gkf` but it may be fully integrated in `gama-local` program later.

In GNU Gama YAML documents are based on four main nodes

```
defaults:

description:

points:

observations:
```

Where `defaults` and `description` are optional and `points` and `observations` are mandatory and each can be used only once. The order of the nodes is arbitrary.

Lets start with a full example

```
defaults:
  sigma-apr : 5
  conf-pr:    0.95

description: >-
  Example: a simple network

points:
  - id:  1783
    y:   453500.000
    x:   104500.000
    adj: xy

  - id:  2044
    y:   461000.000
    x:   101000.000
    fix: xy

observations:
  - from: 1783
    obs:
      - type:  direction
        to:    776
        val:   29.51661
        stdev: 2.0
```

```
        - type:  direction
          to:    351
          val:   94.22790
          stdev: 2.0

    - from: 351
      obs:
        - type:  direction
          to:    2044
          val:   170.48370
          stdev: 2.0

        - type:  distance
          to:    1783
          val:   5522.668
          stdev: 10.0

    - from: 462
      obs:
        - type:  direction
          to:    2505
          val:   299.99973
          stdev: 2.0
```

The `description` node is clearly the simplest one, it just describes a simple text attached to the data. But still there may be a catch. If the description contains *colon* (:), it might confuse the YAML parser because it would be interpreted as a syntax construction. To *escape* colon(s) in the description node we use `>-` to prevent colons to be interpreted as a syntax construction. Always using `>-` with `description` is a safe bet.

The data structure of the YAML document is defined by *indentation*, this principle was inspired by Python programming language, where indentation is very important; Python uses indentation to indicate a block of code.

Practically all attribute names used in out YAML format are the same as in XML data format.

Lets have a look on some more examples. Within `observations:` section we can define height differerence (another kind of a measurement).

```
observations:

  - height-differences:
      - dh:
          from: A
          to  : B
          val : 25.42
          dist: 18.1     # distance in km
      - dh:
          from: B
```

```
          to:   C
          val:  10.34
          dist:  9.4
```

Two remaining observation types are `vectors` and `coordinates`.

```
observations:
  - vectors:
    - vec:
        from: A
        to: S
        dx: 60.0070
        dy: 35.0053
        dz: 54.9953
    - vec:
        from: B
        to: S
        dx: -39.9974
        dy: 34.9928
        dz: 54.9976
```

and

```
observations:
  - coordinates:
    - id: 403
      x: 1054612.59853
      y: 644373.60446
    - id: 407
      x: 1054821.17131
      y: 644025.97479

....

    - cov-mat:
        dim: 20
        band: 19
        upper-part:
          6.7589719e+01 1.8437742e+01 1.3176856e+01 ...
```

Typically any observation set can define its covariance matrix.

You may wish to compare YAML and XML data files available from Gama tests suite in tests/gama-local/input directory (files *.gkf and *.yaml).

The gama-local input xml data can be formally validated against the XSD definition. Unfortunatelly there is no formal definition of YAML input. Within the testing suite of GNU Gama project we have a test that validates all available YAML files converted to XML by the formal XSD definition, see the test `xmllint-gama-local-yaml2gkf`.

3.1 YAML support

GNU Gama YAML input format is dependent on C++ YAML-CPP library written by Jesse Beder *https://yaml.org/*. With the Gama primary build system (autotools) you need to install the library at your system, for example on Debian like systems it is libyaml-cpp-dev package.

A different solution is used in the alternative Gama cmake based build, where the source codes are expected to be available from the `lib` directory. Change to "*GNU Gama sources*"/lib and clone the git repository.

```
cd "GNU Gama sources"/lib
git clone https://github.com/jbeder/yaml-cpp
```

4 SQL schema, SQLite and `gama-local`

The input data for a local geodetic network adjustment (program `gama-local`) can be strored in SQLite 3 database file. The general information about SQLite can be found at

`http://www.sqlite.org/`

Input data (points, observations and other related information) are stored in SQLite database file. Native SQLite C/C++ API is used for reading SQLite database file. It is described at

`http://www.sqlite.org/c3ref/intro.html`

Please note if you compile GNU Gama as described in Section 1.2 [Install], page 2, and SQLite library is not installed on your system, GNU Gama would be compiled without SQLite support.

SQL schema (`CREATE` statements) is in `gama-local-schema.sql` file which is part of GNU Gama distribution and is in the `xml` directory.

All tables for `gama-local` are prefixed with `gnu_gama_local_`. In the documentation table names are referred without this prefix. For example table `gnu_gama_local_points` is referred as `points`.

Database scheme used for SQLite database is also valid in other SQL database systems. Almost every column has some constraint to ensure correctness.

You can convert existing XML input file to SQL commands with program `gama-local-xml2sql`, for example

```
$ gama-local-xml2sql geodet-pc geodet-pc-123.gkf geodet-pc.sql
```

4.1 Working with SQLite database

First of all you have to create tables for GNU Gama in SQLite database file (here with `db` extension, but you can choose your own, e.g. `sqlite`).

```
$ sqlite3 gama.db < gama-local-schema.sql
```

You can check created tables by following commands (fist in command line, second in SQLite command line).

```
$ sqlite3 gama.db
sqlite> .tables
```

Output should look like this:

```
gnu_gama_local_clusters        gnu_gama_local_descriptions
gnu_gama_local_configurations  gnu_gama_local_obs
gnu_gama_local_coordinates     gnu_gama_local_points
gnu_gama_local_covmat          gnu_gama_local_vectors
```

When you have created tables you can import data. One way is to process file with SQL statements.

```
$ sqlite3 gama.db < geodet-pc.sql
```

Another way can be filing database file in another program.

For using `sqlite3` command you need a command line interface for SQLite 3 installed on your system (e.g. `sqlite3` package).

4.2 Units in SQL tables

In the `gama-local` SQLite database, distances are given in meters and their standard deviations (rms errors) in millimeters. Angular values are given in radians as well as their standard deviations.

Conversions between radians, gons and degrees:

$$\text{rad} = \text{gon} \cdot \frac{\pi}{200} = \text{deg} \cdot \frac{\pi}{180}$$

4.3 Network SQL definition

Network definitions are stored in the `configurations` table. This table contains all parameters for each network such as value of a priori reference standard deviation or orientation of the `xy` orthogonal coordinate system axes.

It is obvious that in one database file can be stored more networks (configurations).

Configuration descriptions (annotation or comments) are stored separately in table `descriptions`. The description is split to many records because of compatibility with various databases (not all databases implements type `TEXT`).

Field (attribute) `conf_id` identifies a configuration in the database. Field `conf_name` is used to identify configuration outside the database (e.g. parameter in command-line when reading data from database to `gama-local`).

Table `configurations` contains all parameters specified in tag `<parameters />` (see Section 2.5 [Network parameters], page 11) and also `gama-local` command line parameters (see Section 1.3 [Program gama-local], page 3). The list of all table attributes (parameters) follows.

- `sigma_apr` value of a priori reference standard deviation—square root of reference variance (default value 10)
- `conf_pr` confidence probability used in statistical tests (dafault value 0.95)
- `tol_abs` tolerance for identification of gross absolute terms in project equations (default value 1000 mm)
- `sigma_act` actual type of reference standard deviation use in statistical tests (`aposteriori | apriori`); default value is `aposteriori`
- `update_cc` enables user to control if coordinates of constrained points are updated in iterative adjustment. If test on linerarization fails (see Section 5.9 [Linearization], page 47), Gama tries to improve approximate coordinates of adjusted points and repeats the whole adjustment. Coordinates of constrained points are implicitly not changed during iterations. Acceptable values are `yes`, `no`, default value is `yes`.
- `axes_xy` orientation of axes `x` and `y`; value `ne` implies that axis `x` is oriented north and axis `y` is oriented east. Acceptable values are `ne`, `sw`, `es`, `wn` for left-handed coordinate systems and `en`, `nw`, `se`, `ws` for right-handed coordinate systems (default value is `ne`).
- `angles right-handed` defines counterclockwise observed angles and/or directions, value `left-handed` defines clockwise observed angles and/or directions (default value is `left-handed`).
- `epoch` is measurement epoch. It is floating point number (default value is `0.0`).

- `algorithm` specifies numerical method used for solution of the adjustment. For Singular Value Decomposition set value to `svd`. Value `gso` stands for block matrix algorithm GSO by Frantisek Charamza based on Gram-Schmidt orthogonalization, value `cholesky` for Cholesky decomposition of semidefinite matrix of normal equations and value `envelope` for a Cholesky decomposition with *envelope* reduction of the sparse matrix. Default value is `svd`.
- `ang_units` Angular units of angles in `gama-local` output. Value `400` stands for gons and value `360` for degrees (default value is `400`). Note that this doesn't effect units of angles in database. For further information about angular units see Section 2.1 [Angular units], page 9.
- `latitude` is mean latitude in network area. Default value is `50` (gons).
- `ellipsoid` is name of ellipsoid (see Section 6.2 [Supported ellipsoids], page 53).

All fields are mandatory except `ellipsoid` field. For additional information about handling geodetic systems in `gama-local` see Section 2.3 [Network definition], page 10.

Example (`configuration` table contents):

```
conf_id|conf_name|sigma_apr|conf_pr|tol_abs|sigma_act  |update_cc|...
---------------------------------------------------------------------
1      |geodet-pc|10.0     |0.95   |1000.0 |aposteriori|no       |...

... axes_xy|angles      |epoch|algorithm|ang_units|latitude|ellipsoid
---------------------------------------------------------------------
... ne     |left-handed|0.0  |svd      |400      |50.0    |
```

The list of `description` table attributes follows.

- `conf_id` is id of configuration which description (text) belongs to.
- `id` identifies text in a database.
- `text` is part of configuration description. Its SQL type is `VARCHAR(1000)`.

There can be more than one text for one configuration. All texts related to one configuration are concatenated to one description.

Example (`description` table contents):

```
conf_id|indx|text
---------------------------------------------
1      |1   |Frantisek Charamza: GEODET/PC, ...
```

4.4 Table `points`

- `conf_id` is id of configuration which points belongs to.
- `id` identifies point in a database and also in an output. It is mandatory and it is character string (SQL type is `VARCHAR(80)`). Point `id` has to be unique within one configuration. In documentation it is referred as point identification or point id.
- `x`, `y` and `z` coordinates of a point. Coordinate `z` is considered as height.
- `txy` and `tz` specify the type of coordinates `x`, `y` and `z`. Acceptable values are `fixed`, `adjusted` and `constrained` (there is no default value). For details see Section 2.7 [Points], page 12.

Example (table contents):

```
conf_id|id |x       |y      |z|txy     |tz
------------------------------------------
1      |201|78594.91|9498.26| |fixed   |
1      |205|78907.88|7206.65| |fixed   |
1      |206|76701.57|6633.27| |fixed   |
1      |207|        |       | |adjusted|
```

4.5 Table `clusters`

The cluster is a group of observations with the common covariance matrix. The covariance matrix allows to express any combination of correlations among observations in cluster (including uncorrelated observations, where covariance matrix is diagonal). For explanation see Section 6.1 [Observation data and points], page 51.

In the database observations are stored in three tables: `obs`, `coordinates` and `vectors`. Cluster's covariance matrix is stored in table `covmat`. Every observation, vector or coordinate in database has to be in some cluster.

- `conf_id` is id of configuration which cluster belongs to.
- `ccluster` identifies a cluster within one configuration.
- `dim` and `band` specify dimension and bandwidth of covariance matrix. The bandwidth of the diagonal matrix is equal to 0 and a fully-populated covariance matrix has a bandwidth of `dim-1` (`band` maximum possible value is `dim-1`).
- `tag` specifies type of observations in cluster which also implies the table where they are stored in. `obs` and `height-differences` stand for `obs` table, `coordinates` and `vectors` stand for `coordinates` table and `vectors` table respectively.

Observations, vectors and coordinates are identified by configuration id (`conf_id`), cluster id `ccluster` and theirs index (`indx`). Observation index (`indx`) has to be unique within observations of one cluster (which belongs to one configuration). The same applies for vectors and coordinates.

See also Section 2.8 [Set of observations], page 13.

Example (table contents):

```
conf_id|ccluster|dim|band|tag
-----------------------------
1      |1       |3  |0   |obs
1      |4       |4  |0   |obs
```

4.6 Table `covmat`

Values of cluster covariance matrix are stored in `covmat` table. Attributes `conf_id`, `ccluster` identifies covariance matrix. Value position in matrix is specified by `rind` and `cind` fields.

- `conf_id` is id of configuration which cluster belongs to.
- `ccluster` is id of cluster which matrix belongs to.
- `rind` is row number in covariance matrix

- `cind` is column number covariance matrix
- `val` is value itself (variance or covariance).

Values `rind` and `cind` have to respect `dim` and `band` specified in table `clusters`. If value in covariance matrix is not specified (record is missing), it is considered to be zero.

Example (table contents):

```
conf_id|ccluster|rind|cind|val
--------------------------------
1      |1       |1   |1   |400.0
1      |1       |2   |2   |400.0
1      |1       |3   |3   |400.0
1      |4       |1   |1   |400.0
1      |4       |2   |2   |400.0
1      |4       |3   |3   |400.0
1      |4       |4   |4   |400.0
```

4.7 Table `obs`

Table `obs` contains simple observations like direction or distance.

- `conf_id` is id of configuration which cluster belongs to.
- `ccluster` is id of cluster which observation belongs to.
- `indx` identifies observation within cluster. It has to be positive integer.
- `tag` specifies a type of an observation. Allowed `tag`s follows.
  - `direction` for directions.
  - `distance` for horizontal distances.
  - `angle` for angles.
  - `s-distance` for slope distances (space distances).
  - `z-angle` for zenith angles.
  - `azimuth` for azimuth angles.
  - `dh` for leveling height differences.
- `from_id` is stand point identification. It is mandatory and it must not differ within one cluster for observations with `tag = 'direction'` .
- `to_id` is target identification (mandatory).
- `to_id2` is second target identification. It is valid and mandatory only for angles (`tag = 'angle'`).
- `val` is observation value. It is mandatory for all observation types.
- `stdev` is value of standard deviation. It is used when variance in covariance matrix is not specified.
- `from_dh` is value of instrument height (optional).
- `to_dh` is value of reflector/target height (optional).
- `to_dh2` is value of second reflector/target height (optional). It is valid only for angles.
- `dist` is distance of leveling section. It is valid only for height-differences (`tag = 'dh'`).

- `rejected` specifies whether observation is rejected (passive) or not. Value `0` stand for not rejected, value `1` for rejected. It is mandatory. Default value is `0`.

Example (table contents without empty columns):

```
conf_id|ccluster|indx|tag      |from_id|to_id|val           |rejected
---------------------------------------------------------------------
1      |1       |1   |direction|201    |202  |0.0           |0
1      |1       |2   |direction|201    |207  |0.817750284544|0
1      |1       |3   |direction|201    |205  |2.020073921388|0
```

4.8 Table `coordinates`

Table `coordinates` contains control (known) coordinates.

- `conf_id` is id of configuration which cluster belongs to.
- `ccluster` is id of cluster which coordinates belongs to.
- `indx` identifies coordinates within cluster. It has to be positive integer.
- `id` is point identification.
- `x`, `y` and `z` are coordinates.
- `rejected` specifies whether observation is rejected (passive) or not. Value `0` stand for not rejected, value `1` for rejected. Default value is `0`.

See also Section 2.16 [Control coordinates], page 19.

4.9 Table `vectors`

Table `vectors` contains coordinate differences (vectors).

- `conf_id` is id of configuration which cluster belongs to.
- `ccluster` is id of cluster which vector belongs to.
- `indx` identifies vector within cluster. It has to be positive integer.
- `from_id` is point identification. It identifies initial point.
- `to_id` is point identification. It identifies terminal point.
- `dx`, `dy` and `dz` are coordinate differences.
- `from_dh` is value of initial point height. It is optional.
- `to_dh` is value of terminal point height. It is optional.
- `rejected` integer default 0 not null,

See also Section 2.17 [Coordinate differences], page 19.

4.10 Example of local geodetic network in SQL

Providing complete example would be reasonable because of its extent. However, you can obtain example by following these instructions:

Create a file with XML representation of network by copy and paste example from Section 2.19 [Example], page 20, to a new file. Note that file should start with `<?xml version="1.0" ?>` (no whitespace). Alternatively you can use existing XML file from collection of sample networks (see Section 1.1 [Download], page 2). Then you can convert your XML file (here `example_network.xml`) to SQL statements by program `gama-local-xml2sql` (the path depends on your Gama installation).

```
$ gama-local-xml2sql example_net example_network.xml example_network.sql
```

Now you have example network (configuration `example_net`) in the form of SQL `INSERT` statements in the file `example_network.sql`.

Another representations you can create and fill SQLite database (for details see Section 4.1 [Working with SQLite database], page 29):

```
$ sqlite3 examples.db < gama-local-schema.sql
$ sqlite3 examples.db < example_network.sql
$ sqlite3 examples.db
```

Once you have SQLite database, you can work with it from SQLite command line. You can get nice output by executing following commands.

```
sqlite> .mode column
sqlite> .nullvalue NULL
sqlite> SELECT * FROM gnu_gama_local_configurations;
sqlite> SELECT * FROM gnu_gama_local_points;
sqlite> SELECT * FROM gnu_gama_local_clusters;
sqlite> SELECT * FROM gnu_gama_local_covmat;
sqlite> SELECT * FROM gnu_gama_local_obs;
```

Or you can get database dump (`CREATE` and `INSERT` statements) by

```
sqlite> .dump
```

If it is not enough for you, you can try one of GUI tools for SQLite.

5 Network adjustment with `gama-local`

Adjustment of local geodetic network is a classical case of *adjustment of indirect observations.* After estimation of approximate values of unknown parameters (coordinates of points) and linearization of functions describing relations between observations and parameters we solve linear system of equations

$$\mathbf{A}\mathbf{x} = \mathbf{b} + \mathbf{v}, \tag{1}$$

where $\mathbf{A}$ is coefficient matrix, $\mathbf{b}$ is vector of absolute terms (right hand side) and $\mathbf{v}$ is vector of residuals. This system is (generally) overdetermined and we seek the solution $\mathbf{x}$ satisfying the basic criterion of Least Squares

$$\mathbf{v}'\mathbf{P}\mathbf{v} = \min, \tag{2}$$

where $\mathbf{P}$ is weight matrix. This criterion unambiguously defines the shape of adjusted network.

In the case of *free network* the system (1) is singular (matrix $\mathbf{A}$ has linearly dependent columns) and we have to define second regularization criterion

$$\sum_{i\in\Omega} x_i^2 = \min, \tag{3}$$

stating that at the same time we demand that the sum of squares corrections of selected parameters is minimal (corrections of unknown parameters with indexes from the set Ω). Geometrically this criterion is equivalent to adjustment of the network according to (2) with simultaneous transformation to the selected set of fiducial points. This transformation does not change the shape of adjusted network.

Often it is advantageous to work with a *homogenized system,* ie. with the system of project equations in which coefficient of each row and absolute term are multiplied by square root of the weight of corresponding observation.

$$\tilde{\mathbf{A}}\mathbf{x} = \tilde{\mathbf{b}}, \tag{4}$$

where $\tilde{\mathbf{A}} = \mathbf{P}^{1/2}\mathbf{A}$, $\tilde{\mathbf{b}} = \mathbf{P}^{1/2}\mathbf{b}$. Symbol $\mathbf{P}^{1/2}$ denotes diagonal matrix of square roots of observation weights (or Cholesky decomposition of covariance matrix in the case of correlated observations). To criterion (2) corresponds in the case of homogenized system criterion

$$\tilde{\mathbf{v}}'\mathbf{v} = \min\,. \tag{5}$$

Normal equations are clearly equivalent for both systems.

$$(\mathbf{A}'\mathbf{P}\mathbf{A})\mathbf{x} = (\mathbf{A}'\mathbf{P}\mathbf{b}) \quad\equiv\quad (\tilde{\mathbf{A}}'\tilde{\mathbf{A}})\mathbf{x} = (\tilde{\mathbf{A}}\tilde{\mathbf{b}}).$$

Between weight coefficients of the original system (1) and homogenized system (4) are the following relations

$$\begin{aligned}
q_{x_i} &= \tilde{q}_{x_i}, \qquad i = 1,\ldots,n,\\
q_{L_j} &= \tilde{q}_{L_j}/p_j, \qquad j = 1,\ldots,m,\\
q_{v_k} &= \tilde{q}_{v_k}/p_k = (1-\tilde{q}_{L_k})/p_k = 1/p_k - q_{L_k}, \qquad k = 1,\ldots,m.
\end{aligned}$$

5.1 Approximate coordinates

For computation of coefficients in system (1) (ie. during linearization) we need, first of all, an estimate of approximate coordinates of points and approximate values of orientations of observed directions sets.

Approximate values of unknown parameters are usually not known and we have to compute them from the available observations. For approximate value of orientation program `gama-local` uses median of all estimates from the given set of directions to the points with known coordinates. Median is less sensitive to outliers than arithmetic mean which is normally used for approximate estimate of orientations

During the phase of computation of approximate coordinate of points, program `gama-local` walks through the list of computed points and for each point gathers all determining elements pointing to points with known or previously computed coordinates. Determining elements are

outer bearing (oriented half-line) starting from the point with known coordinates and pointing to the computed point

distance between given and computed points

inner angle with vertex in the computed point and arms intersecting given points

For all combinations of determining elements program `gama-local` computes intersections and estimates approximate coordinates as the median of all available solutions.

If at least one point was resolved while iterating through the list, the whole cycle is repeated.

If no more coordinates can be solved using intersections and points with unknown coordinates are remaining, program tries to compute coordinates of unresolved points in a local coordinates system and obtain their coordinates using similarity transformation. If a transformation succeeds to resolve coordinates at least one computed point and there are still some points without coordinates left, the whole process is repeated. Classes for computation of approximate coordinates have been written by Jiri Vesely.

If program `gama-local` fails to compute approximate coordinates of some of the network points, they are eliminated from the adjustment and they are listed in the output listing.

With the outlined strategy, program `gama-local` is able to estimate approximate coordinates in most of the cases we normally meet in surveying profession. Still there are cases in which the solution fails. One example is an inserted horizontal traverse with sets of observed direction on both ends but without a connecting observed distance. The solution of approximate coordinates can fail when there is a number of gross error for example resulting from confusion of point identifications but in normal situations, leaving computation of approximate coordinates on program `gama-local` is recommended.

Example

```
Computation of approximate coordinates of points
************************************************

Number of points with given coordinates:      2
Number of solved points                :      2
Number of observations                 :      4
```

```
-----------------------------------------------------
Successfully solved points             :      0
Remaining unsolved points              :      2


List of unresolved points
*************************
422
424
```

5.2 Gross absolute terms

One of parameters in XML input of program `gama-local` is tolerance `tol-abs` for detecting of gross absolute terms in project equations. Observations with outlying absolute terms are always excluded from adjustment.

For measured distances program tests difference between observed value d_i and distance computed from approximate coordinates d_0

$$|d_i - d_0| > \texttt{tol} - \texttt{abs},$$

for observed directions program `gama-local` tests transverse deviation corresponding to absolute term b_i from project equations (1)

$$|b_i| d_0 > \texttt{tol} - \texttt{abs}$$

and similarly for angles, program tests the greater of two deviations corresponding to left and right distances (left and right arm of the angle)

$$|b_i| \max\{d_{0_l}, d_{0_r}\} > \texttt{tol} - \texttt{abs}.$$

Default value of parameter `tol-abs` is 1000 mm.

Example

```
Outlying absolute terms in project equations
********************************************

   i   standpoint       target           observed     absolute
=========================================== value ===== term ==

   2          103          104 dir.    301.087900       -9989.1


Observations with outlying absolute terms removed
```

5.3 Parameters of statistical analysis

Program `gama-local` uses two basic statistical parameters

- confidence probability P (default value is 95%, see input XML parameter `conf-pr`) and
- actual type of reference standard deviation m_{0a} (parameter `sigma-act`).

Confidence probability determines significance level on which statistical tests of adjusted quantities are carried. Actual type of reference standard deviation m_{0a} specifies whether during statistical analysis we use an a priori reference standard deviation m_0 or an a posteriori estimate $m_0^{'}$. On the type of actual reference standard deviation depends the choice of density functions of stochastic quantities in statistical analysis of the adjustment.

> **A priori reference standard deviation** m_0 is an estimate of the standard deviation of an observation with the unit weight. Numerically it is a scaling factor used in calculation of the weights. If we change m_0, only the sum of weighted residuals squares is changed and all adjustment results remain the same (there is just one least squares solution). m_0 can be selected in cases when we know its value in advance and with sufficient reliability. Another situation when m_0 is used are networks with low number of degrees of freedom (poorly overdetermined systems) or when veen degrees of freedom is zero. Examples may be analysis of network models etc.
>
> **A posteri estimate of reference standard deviation** $m_0^{'}$ is used in cases when a priori value of reference standard deviation m_0 is not known and when degrees of freedom is sufficiently high and reliable for empirical estimate of $m_0^{'}$.

The standard deviation of an adjusted quantity θ is computed in dependece on the choice of actual type of reference standard deviation m_{0a} according to formula

$$m_{\theta_i} = m_{0a}\sqrt{q_{\theta_{ii}}},$$

where $q_{\theta_{ii}}$ is weight coefficient (cofactor) of the i-th adjusted unknown parameter (coordinate or orientation, $\theta = x_i$) or i-th adjusted observation (distance, direction, $\ldots, \theta = L_i$).

Apart from standard deviation m_θ, program `gama-local` computes for adjusted quantity θ its *confidence interval* (Θ_1, Θ_2) in which the real value Θ is located with probability P

$$P(\Theta_1 < \Theta < \Theta_2) = P,$$

$$\Theta_1 = \theta - k_p m_\theta, \qquad \Theta_2 = \theta + k_p m_\theta,$$

where coefficient k_p depends on confidence probability P and in the case of low number of degrees of freedom on the choice of actual type of reference standard deviation m_{0a}.

Coefficient k_p is computed for $m_{0a} = m_0$ as critical value of normal distribution for probability $\alpha/2$, for the case of choice $m_{0a} = m_0^{'}$ as critical value of Student distribution on confidence level $\alpha/2$ with τ degrees of freedom

$$k_p = \begin{cases} u_{\alpha/2} & \text{if } m_{0a} = m_0, \\ t_{\alpha/2,\tau} & \text{if } m_{0a} = m_0^{'}. \end{cases}$$

Similarly confidence ellipses for adjusted points are defined in the following text.

5.4 Test on the reference standard deviation

Null hypothesis $H_0 : m_0 = m_0^{'}$ is tested versus alternative hypothesis $H_1 : m_0 \neq m_0^{'}$. Test criterion is ratio of a posteriori estimate of reference standard deviation

$$m_0^{'} = \sqrt{\mathbf{v}'\mathbf{P}\mathbf{v}/\tau}$$

and a priori reference standard deviation m_0 (input data parameter `m0-apr`). For given significance level α lower and upper bounds of interval (L, U) are computed so, that if hypothesis H_0 is true, probabilities $P(m_0^{'}/m_0 \leq D)$ and $P(m_0^{'}/m_0 \geq H)$ are equal to $\alpha/2$. Lower and upper bounds of the interval are computed as

$$L = \sqrt{(\chi^2_{1-\alpha/2,\tau}/\tau)}, \qquad U = \sqrt{(\chi^2_{\alpha/2,\tau}/\tau)}.$$

Probability

$$P(L < m_0^{'}/m_0 < U) = \texttt{conf} - \texttt{pr}$$

is by default 95%, this corresponds to 5% confidence level test.

Exceeding the upper limit H of the confidence interval can be caused even by a single gross error (one outlying observation). Method of Least Squares is generally very sensitive to presence of outliers. Safely can be detected only one observation whose elimination leads to maximal decrease of a posteriori estimate of reference standard deviation

$$m_0^{''} = \sqrt{(\mathbf{v}'\mathbf{P}\mathbf{v} - \delta)/(\tau - 1)}, \qquad \delta = \max(v_i^2/q_{v_i}), \tag{6}$$

where

$$q_{v_i} = 1/p_i - q_{L_i} \tag{7}$$

is weight coefficient of i -th residual. If the set of observations contains only one gross error, the outlying observation is likely to be detected, but this can not be guaranteed.

In addition, program `gama-local` computes a posteriori estimate of reference standard deviation separately for horizontal distances and directions and/or angles after formula from

$$m_{0t}' = \sqrt{\sum \tilde{v}_{i_t}^2 / \sum \tilde{q}_{v_{i_t}}}, \qquad t = d, s,$$

where symbol t denotes observed distances, directions and/or angles.

Example

```
m0  apriori  :    10.00
m0' empirical:     9.64          [pvv] : 3.43560e+03

During statistical analysis we work

- with empirical standard deviation 9.64
- with confidence level            95 %

Ratio m0' empirical / m0 apriori: 0.964
95 % interval (0.773, 1.227) contains value m0'/m0
```

```
m0’/m0 (distances): 0.997   m0’/m0 (directions): 0.943

Maximal decrease of m0’’/m0 on elimination of one observation: 0.892

Maximal studentized residual 2.48 exceeds critical value 1.95
on significance level 5 % for observation #35
<distance from="407" to="422" val="346.415" stdev="5.0" />
```

5.5 Information on points

Program `gama-local` lists separately review of coordinates of fixed and adjusted points; adjusted *constrained* coordinates are marked with `*`; see equation (3). Adjusted coordinate standard deviations m_x and m_y, and values for computing confidence intervals are given in the listing of adjusted coordinates (Section 5.3 [Statistical analysis], page 40). In the review index i is the index of unknown x_i from the system of project equations (1) corresponding to the point coordinates x and y.

Example

```
Fixed points
************

       point         x              y
========================================

           1  1054980.484     644498.590
           2  1054933.801     643654.101


Adjusted coordinates
********************

  i        point    approximate  correction  adjusted    std.dev conf.i.
===================== value ====== [m] ====== value ========== [mm] ===

             422
  2            x   1055167.22747  -0.00510 1055167.22237     2.7     5.4
  3            y    644041.46119   0.00023  644041.46142     2.5     5.1

             424
  4            X * 1055205.41198  -0.00056 1055205.41142     3.1     6.3
  5            Y *  644318.24425  -0.00125  644318.24300     3.6     7.2
```

For adjusted points, program summarizes information on standard ellipses, confidence ellipses, mean square positional errors (m_p), mean coordinate errors (m_{xy}) and coefficients g characterizing position of approximate coordinates with regard to the confidence ellipse.

Example

```
Mean errors and parameters of error ellipses
********************************************

  point     mp      mxy      mean error ellipse  conf.err. ellipse   g
========== [mm] == [mm] ==== a [mm] b  alpha[g] ==== a' [mm] b' ========

    422     3.6     2.6     2.7     2.5   187.0     6.8      6.4     0.8
    424     4.7     3.4     3.7     2.9   131.8     9.5      7.4     0.2
    403     5.7     4.0     4.3     3.6    78.9    11.0      9.3     1.1
```

Mean square positional error m_p and mean coordinate error (m_{xy}) are computed as

$$m_p = \sqrt{m_y^2 + m_x^2}, \qquad m_{xy} = m_p/\sqrt{2},$$

where m_y^2 and m_x^2 are squares of standard deviations (variances) of adjusted points coordinates.

Semimajor and semiminor axes of standard ellipse are denoted as a and b in the listing, bearing of semimajor axis is denoted as α and they are computed from covariances of adjusted coordinates

$$a = \sqrt{\frac{1}{2}(\mathrm{cov}\ yy + \mathrm{cov}\ xx + c)}, \qquad b = \sqrt{\frac{1}{2}(\mathrm{cov}\ yy + \mathrm{cov}\ xx - c)},$$

$$c = \sqrt{(\mathrm{cov}\ xx - \mathrm{cov}\ yy)^2 + 4(\mathrm{cov}\ xy)^2},$$

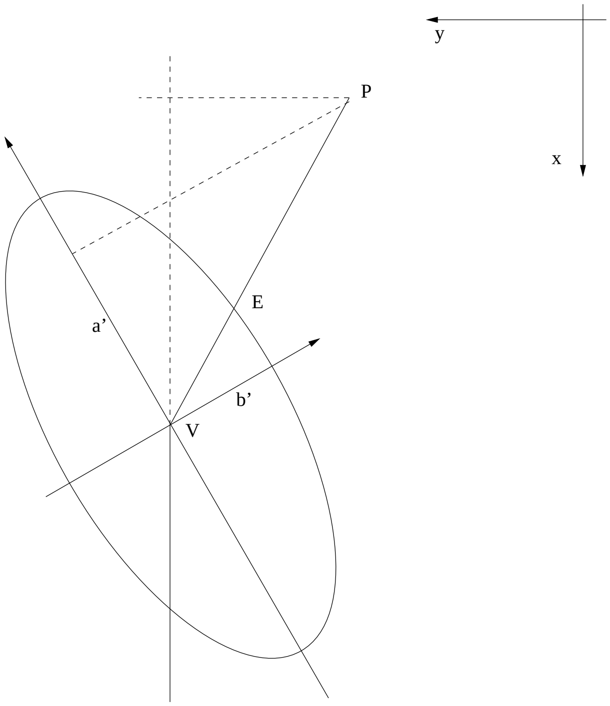
$$\tan 2\alpha = 2(\mathrm{cov}\ xy)/(\mathrm{cov}\ xx - \mathrm{cov}\ yy).$$

The angle α (the bearing of semimajor axis) is measured clockwise from X axis.

Probability that standard ellipse covers real position of a point is relatively low. For this reason program `gama-local` computes extra *confidence ellipse* for which the probability of covering real point position is equal to the given confidence probability. Both ellipsy are located in the same center, they share the same bearing of semimajor axes and they are similar. For lengths of their semi-axis holds

$$a' = k_p a, \qquad b' = k_p b,$$

where k_p is a coefficient computed for the given probability P as defined in Section 5.3 [Statistical analysis], page 40.



Position of approximate coordinates of an adjusted point with respect to its confidence ellipse is described by two points P and V where point P depicts approximate coordinates and V adjusted coordinates. Point E is the intersection of oriented half-line VP and the confidence ellipse. Coefficient g is defined as the ration of abscissae

$$g = \overline{VP}/\overline{VE}.$$

Three cases are possible

- $g < 1$ approximate coordinates of adjusted point are located inside the confidence ellipse
- $g = 1$ approximate coordinates of adjusted point are located on the confidence ellipse
- $g > 1$ approximate coordinates of adjusted point are outside the confidence ellipse

The coefficient g is calculated from formula

$$g = \sqrt{(a_0/a')^2 + (b_0/b')^2}$$

where

$$b_0 = \delta_y \cos\alpha - \delta_x \sin\alpha, \qquad a_0 = \delta_y \sin\alpha - \delta_x \cos\alpha$$

symbol δ is used for correction of approximate coordinates and α is bearing of confidence ellipse semimajor axis.

If network contains sets of observed directions, program writes information on corresponding adjusted orientations, standard deviations and confidence intervals. Index i is the same as in the case of adjusted coordinates the index of i -th adjusted unknown in the project equations.

Example

```
Adjusted bearings
*****************

  i   standpoint   approximate  correction  adjusted   std.dev conf.i.
==================== value [g] ==== [g] === value [g] ======= [cc] ===

  1            1    296.484371  -0.000917  296.483454      5.1    10.3
 10            2     96.484371   0.000708   96.485079      5.1    10.4
 21          403     20.850571  -0.001953   20.848618      8.8    17.7
```

5.6 Adjusted observations and residuals

In the review of adjusted observations program `gama-local` prints index of the observation, index of the row in matrix **A** in the system (1), identifications of standpoint and target point, type of the observation, its approximate and adjusted value, standard deviation and confidence interval.

Example

```
Adjusted observations
*********************

   i   standpoint   target         observed     adjusted std.dev conf.i.
===================================== value ==== [m|g] ====== [mm|cc] ==

   1            1        2 dis.   845.77700    845.77907     3.0     6.1
   2                   422 dir.   28.205700    28.205613     5.1    10.3
   3                   424 dir.   60.490600    60.491359     6.7    13.6
```

Review of residuals serves for analysis of observations and containts values of normalized or studentized residuals (depending on type of m_{0a} used) and three characteristics. Theese are coefficient `f` identifying weak network elements and estimates of real error of observation `e-obs` and real error of its adjusted value `e-adj`, see definition in the following text.

If normalized or studentized residual exceeds critical value for the given confidence probability, it is marked in the review with symbol `c` (critical) and maximal normalized or studentized residual is marked with symbol `m`.

Example

```
Residuals and analysis of observations
**************************************

   i standpoint   target        f[%]         v    |v'|     e-obs.  e-adj.
======================================== [mm|cc] =========== [mm|cc] ===

   1          1        2 dir.   47.4       9.170  1.1       12.7    3.5
```

```
   2                   422 dir.   47.0      -0.873  0.1       -1.2   -0.3
   3                   424 dir.   30.3       7.588  1.1       14.8    7.2
```

5.7 Identification of weak network elements

When planning observations in a geodetic network we always try to guarantee that all observed elements are checked by other measurements. Only with redundant measurements it is possible to adjust observations and possibly remove blunders that might otherwise totaly corrupt the whole set of measurements. Apart from sufficient number of redundant observations the degree of control of single observed elements is given by the network configuration, ie. its geometry.

Less controlled observations represent weak network elements and they can in extreme cases even disable detection of gross observational errors as it is in the case of uncontrolled observations. There are two limit cases of observation control

fully controlled observation as is for example an observed distance between two fixed points (standard deviation of the adjusted element is zero; standard deviation of the residual equals to the standard deviation if the observation) and

uncontrolled observations as is a free polar bar for example (standard deviation of adjusted value is equal to standard deviation of observed quantity; residual and standard deviation of the residual are zero).

Weakly controlled or uncontrolled observations can result even from elimination of certain suspisios observations during analysis of adjusment.

Standard deviation of adjusted observations is less than standard deviation of the measurement. Degree of observation control in network is defined as coefficient

$$f = 100\frac{m_\ell - m_L}{m_\ell}, \tag{8}$$

where m_ℓ is standard deviation of observed quantity and m_L is standard deviation computed from a posteriori reference standard deviation m_0 . We consider observed network element to be

uncontrolled if $f < 0.1$ (in listing marked with letter `u`),

weakly controlled if $0.1 \le f < 5$ (in listing marked with letter `w`).

5.8 Estimation of real errors

Acording to previous section we can consider an observation to be controlled if its coefficient $f > 0.1$. Any controlled observation can be eliminated from the network without corrupting the network consistency—network reduced by one controlled observation can be adjusted and all unknown parameters can be compute without the eliminated observation.

Estimate of real error of i -th observation is defined as

$$\varepsilon_{\ell_i} = L_i^{red} - \ell_i, \tag{9}$$

where ℓ_i is value of i -th observation and L_i^{red} is value of i -th network element computed from adjusted coordinates and/or orientations of the reduced network. Similarly is defined the estimate of real error of a residual

$$\varepsilon_{v_i} = L_i^{red} - L_i. \tag{10}$$

Adjustment results are the best statistical estimate of unknown parameters that we have. This holds true even for adjustment of *reduced* network which is not influenced by real error of i -th observation. On favourable occasions differences (9) and (10) can help to detect blunders but to interpret these estimates as *real errors* is possible only with substantial exaggeration. These estimates fail when there are more than one significant observational error. Generally holds tha the weaker the element is controlled in netowrk the less reliable these estimates are.

Estimate of real error of an observation computes program `gama-local` as

$$\varepsilon_{\ell_i} = v_i/(p_i q_{v_i})$$

and estimate of real error of a residual as

$$\varepsilon_{v_i} = \varepsilon_{l_i} - v_i.$$

5.9 Test on linearization

Mathematical model of geodetic network adjustment in `gama-local` is defined as a set of known real-valued differentiable functions

$$\mathbf{L}^* = \varphi(\mathbf{X}^*), \tag{11}$$

where $\mathbf{L}^*$ is a vector of theoretical correct observations and $\mathbf{X}^*$ is a vector of correct values of parameters. For the given sample set of observations $\mathbf{L}$ and the unknown vector of residuals $\mathbf{v}$ we can express the estimate of parameters $\mathbf{X}$ as a nonlinear set of equations

$$\mathbf{L} + \mathbf{v} = \varphi(\mathbf{X}). \tag{12}$$

With approximate values $\mathbf{X_0}$ of unknown parameters

$$\mathbf{X} = \mathbf{X_0} + \mathbf{x}$$

we can linearize the equations (12)

$$\mathbf{L} + \mathbf{v} = \varphi(\mathbf{X_0}) + \left.\frac{\partial\varphi}{\partial\mathbf{X}}\right|_{\mathbf{X}=\mathbf{X_0}} \mathbf{x}$$

yielding the linear set of equations (1) where

$$\mathbf{A} = \left.\frac{\partial\varphi}{\partial\mathbf{X}}\right|_{\mathbf{X}=\mathbf{X_0}} \qquad \text{and} \qquad \mathbf{b} = \mathbf{L} - \varphi(\mathbf{X}_0).$$

Unknown parameters in `gama-local` mathematical model are points coordinates and orientation angles (transforming observed directions to bearings). The observables described by functions (12) belong into two classes

linear observables: horizontal and slope distances, height differences, control coordinates and vectors (coordinate differences),

angular observables: directions, horizontal and zenith angles.

Internally in `gama-local` unknown corrections to linear observables are computed in millimeters and corrections to angular observables in centigrade seconds. To reflect the internal units in used all partial derivatives of angular observables by coordinates are scaled by factor $2000/\pi = 10^{-3} \times (200 \times 10^4/\pi)$.

When computing coefficients of project equations (1) we expect that approximate coordinates of points are known with sufficient accuracy needed for linearization of generally nonlinear relations between observations and unknown paramters. Most often this is true but not always and generally we have to check how close our approximation is to adjusted parameters.

Generally we check linearization in adjustment by double calculation of residuals

$$\begin{aligned} \mathbf{v}^i &= \mathbf{Ax} - \mathbf{b}, \\ \mathbf{v}^{ii} &= \bar{\ell}(\bar{\mathbf{x}}) - \ell, \end{aligned}$$

where in our notation $\mathbf{x}$ is vector of corrections of approximate unknown parameters $\mathbf{x_0}$, $\mathbf{b}$ vector of reduced observations, ℓ vector of observations and $\bar{\ell}(\bar{\mathbf{x}})$ is vector of adjusted observations conputed from adjusted coordiantes $\bar{\mathbf{x}} = \mathbf{x_0} + \mathbf{x}$. Disagreement $\mathbf{v}^i \neq \mathbf{v}^{ii}$ signals discrepancies in linearization.

Program `gama-local` similarly computes and tests differences in values of adjusted observations once computed from residuals and once from adjusted coordinates. For measured directions and angles `gama-local` computes in addition transverse deviation corresponding to computed angle difference in the distance of target point (or the farther of two targets for angle). As a criterion of bad linearization is supposed positional deviation greater or equal to 0.0005 millimetres.

Example

```
Test of linearization error
***************************

Diffs in adj. obs from residuals and from adjusted coordinates
**************************************************************

 i standpoint     target          observed     r        difference
=================================    value  = [mm|cc] = [cc] == [mm]=

 2 3022184030 3022724008 dist.   28.39200    -7.070          -0.003
 3            3022724002 dist.   72.30700   -18.815          -0.001
 7            3000001063 dir.  286.305200    11.272  -0.002  -0.001
 8            3022724008 dir.  357.800600   -23.947   0.037   0.002
```

From the practical point of view it might seem that the tolerance 0.0005 mm for detecting poor linearization is too strict. Its exceeding in program `gama-local` results in repeated adjustment with substitute adjusted coordinates for approximate. Given tolerance was chosen so strict to guarantee that listed output results would never be influenced by linearization and could serve for verification and testing of numerical solutions produced by other programs.

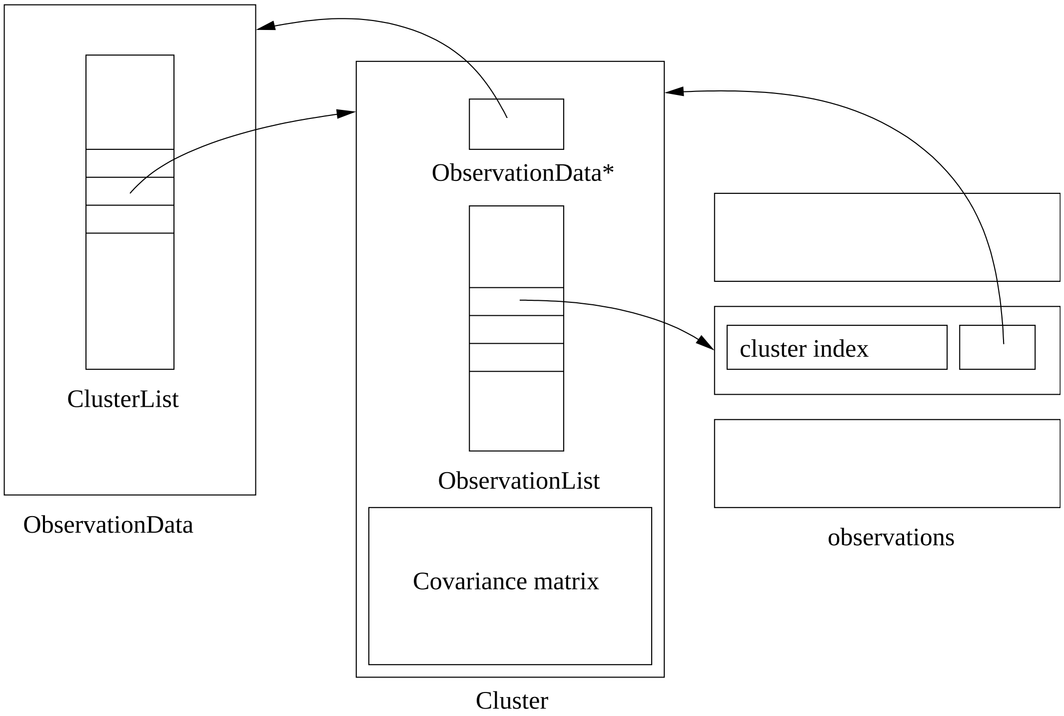
Iterated adjustement with successive improvement of approximate unknown coordinates converges usually even for gross errors in initial estimates of unknown coordinates. If the influence of linearization is detected after adjustment, typically only one iteration is sufficient for recovering.

For any automatically controlled iteration we have to set up certain stopping criterion independent on the convergence and results obtained. Program `gama-local` computes iterated adjustment three times at maximum. If the bad linearization is detected even after three readjustments it signals that given network configuration is somehow suspicious.

6 Data structures and algorithms

6.1 Observation data and points

The Gama observation data structures are designed to enable adjustment of any combination of possibly correlated observations. At its very early stage Gama was limited to adjustment of uncorrelated observations. Only directions and distances were available and observable's weight was stored together with the observed value in a single object. A single array of pointers to observation objects was sufficient for handling all observations. So called *orientation shifts* corresponding to directions measured form a point were stored together with coordinations in *point objects*.



To enable adjustment of possibly correlated observations (like angles derived from observed directions or already adjusted coordinates from a previous adjustment) Gama has come with the concept of *clusters*. Cluster is an object with a common variance-covariance matrix and a list of pointers to observation objects (distances, directions, angles, etc.). Weights were removed from observation objects and replaced with a pointer to the cluster to which the observation belong. All clusters are joined in a common object `ObservationData`; similarly to observations, each cluster contains a pointer to its parent `Observation Data` object. *Orientation shifts* were separated from coordinates and are stored in the cluster containing the bunch of directions and thus number of orientations is not limited to one for a point.

This organisation of observational information has proved to be effective. Template classes `ObservationData` and `Cluster` are used as base classes both in `gama-local` and `gama-g3`

```
template <typename Observation>
  class ObservationData
```

```
  {
  public:
    ClusterList<Observation>  CL;

    ObservationData();
    ObservationData(const ObservationData& cod);
    ~ObservationData();

    ObservationData& operator=(const ObservationData& cod);
    template <typename P> void for_each(const P& p) const;
  };

template <typename Observation>
  class Cluster
  {
  public:
    const ObservationData<Observation>*     observation_data;
    ObservationList<Observation>            observation_list;
    typename Observation::CovarianceMatrix  covariance_matrix;

    Cluster(const ObservationData<Observation>* od);
    virtual ~Cluster();

    virtual Cluster* clone(const ObservationData<Observation>*) const = 0;
    double stdDev(int i) const;
    int size() const;
    void update();
    int  activeCount() const;
    typename Observation::CovarianceMatrix activeCov() const;
  };
```

The following template class `PointBase` for handling point information is used in `gama-g3`. The template class `PointBase` relies internally on `std::map` container but comes with its own interface (in `gama-local` `std::map` was used directly for storing points).

```
template <typename Point>
  class PointBase
  {
    typedef std::map<typename Point::Name, Point*>  Points;

  public:
    PointBase();
    PointBase(const PointBase& cod);
    ~PointBase();

    PointBase& operator=(const PointBase& cod);
    void put(const Point&);
```

```
    void put(Point*);
    Point*       find(const typename Point::Name&);
    const Point* find(const typename Point::Name&) const;
    void erase(const typename Point::Name&);
    void erase();

    class const_iterator;
    const_iterator  begin();
    const_iterator  end  ();

    class iterator;
    iterator  begin();
    iterator  end  ();
  };
```

Template classes `ObservationData` and `PointBase` are defined in namespace `GNU_gama` and are located in the source directory `gnu_gama`.

6.2 Supported ellipsoids

id	a	b, 1/f, f	description	
airy	6377563.396	6356256.910	Airy ellipsoid 1830	[4]
airy_mod	6377340.189	6356034.446	Modified Airy	[4]
apl1965	6378137	298.25	Appl. Physics. 1965	[4]
andrae1876	6377104.43	300.0	Andrae 1876 (Denmark, Iceland)	[4]
australian	6378160	298.25	Australian National 1965	[3]
bessel	6377397.15508	6356078.96290	Bessel ellipsoid 1841	[1]
bessel_nam	6377483.865	299.1528128	Bessel 1841 (Namibia)	[4]
clarke1858a	6378361	6356685	Clarke ellipsoid 1858 1st	[3]
clarke1858b	6378558	6355810	Clarke ellipsoid 1858 2nd	[3]
clarke1866	6378206.4	6356583.8	Clarke ellipsoid 1866	[3]
clarke1880	6378316	6356582	Clarke ellipsoid 1880	[3]
clarke1880m	6378249.145	293.4663	Clarke ellipsoid 1880 (modified)	[4]
cpm1799	6375738.7	334.29	Comm. des Poids et Mesures 1799	[4]
delambre	6376428	311.5	Delambre 1810 (Belgium)	[4]
engelis	6378136.05	298.2566	Engelis 1985	[4]
everest1830	6377276.345	300.8017	Everest 1830	[4]
everest1848	6377304.063	300.8017	Everest 1948	[4]
everest1856	6377301.243	300.8017	Everest 1956	[4]
everest1869	6377295.664	300.8017	Everest 1969	[4]

everest_ss	6377298.556	300.8017	Everest (Sabah and Sarawak)	[4]
fisher1960	6378166	298.3	Fisher 1960 (Mercury Datum)	[3] [4]
fisher1960m	6378155	298.3	Modified Fisher 1960	[3] [4]
fischer1968	6378150	298.3	Fischer 1968	[4]
grs67	6378160	298.2471674270	GRS 67 (IUGG 1967)	[4]
grs80	6378137	298.257222101	Geodetic Reference System 1980	[1]
hayford	6378388	297	Hayford 1909 (International)	[1] [3]
helmert	6378200	298.3	Helmert ellipsoid 1906	[3]
hough	6378270	297	Hough	[4]
iau76	6378140	298.257	IAU 1976	[4]
international	6378388	297	International 1924 (Hayford 1909)	[1] [3]
kaula	6378163	298.24	Kaula 1961	[4]
krassovski	6378245	298.3	Krassovski ellipsoid 1940	[1]
lerch	6378139	298.257	Lerch 1979	[4]
mprts	6397300	191.0	Maupertius 1738	[4]
mercury	6378166	298.3	Mercury spheroid 1960	[3]
merit	6378137	298.257	MERIT 1983	[4]
new_intl	6378157.5	6356772.2	New International 1967	[4]
nwl1965	6378145	298.25	Naval Weapons Lab., 1965	[4]
plessis	6376523	6355863	Plessis 1817 (France)	[4]
se_asia	6378155	6356773.3205	Southeast Asia	[4]
sgs85	6378136	298.257	Soviet Geodetic System 85	[4]
schott	6378157	304.5	Schott 1900 spheroid	[3]
sa1969	6378160	298.25	South American Spheroid 1969	[3]
walbeck	6376896	6355834.8467	Walbeck	[4]
wgs60	6378165	298.3	WGS 60	[4]
wgs66	6378145	298.25	WGS 66	[4]
wgs72	6378135	298.26	WGS 72	[4]
wgs84	6378137	298.257223563	World Geodetic System 1984	[1]

[1] Milos Cimbalnik - Leos Mervart: Vyssi geodezie 1, 1997, Vydavatelstvi CVUT, Praha

[2] Milos Cimbalnik: Derived Geometrical Constants of the Geodetic Reference System 1980, Studia geoph. et geod. 35 (1991), pp. 133-144, NCSAV, Praha

[3] Glossary of the Mapping Sciences, Prepared by a Joint Committe of the American Society of Civil Engineers, American Congress on Surveying and Mapping and American Society for Photogrammetry and Remote Sensing (1994), USA, ISBN 1-57083-011-8, ISBN 0-7844-0050-4

[4] Gerald Evenden: proj - forward cartographic projection filter (rel. 4.3.3), http://www.remotesensing.org/proj

6.3 Transformation from spatial to geographical coordinates

Spatial coordinates (X, Y, Z) can be easily computed from geographical ellipsoidal coordinates (B, L, H), where B is geographical latitude, L geographical longitude and H is elliposidal height, as

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} (N+H)\cos B \cos L \\ (N+H)\cos B \sin L \\ (N(1-e^2)+H)\sin B \end{pmatrix}$$

where $N = a/\sqrt{1 - e^2 sin^2 B}$ is the radius of curvature in the prime vertical, $e^2 = (a^2 - b^2)/a^2$ is the first eccentricity for the given rotational ellipsoid (spheroid) with semi-major axis a and semi-minor axis b.

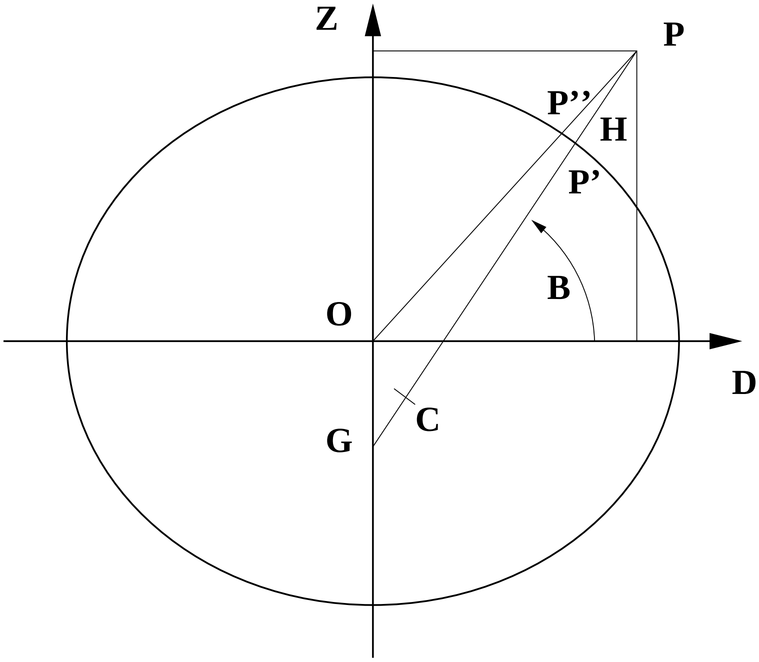
In the case of coordiante transformation from (X, Y, Z) to (B, L, H), the longitude is given by the formula

$$\tan L = Y/X.$$

Now we can introduce $D = \sqrt{X^2 + Y^2}$, so that the cartesian system become (D, Z). Coordinates B and H are then usually computed by iteration with some starting value of B_0, for example $\tan B_0 = Z/D/(1 - e^2)$,

$$\tan B_i = Z/D + \frac{N_{i-1}}{(N_{i-1} + H_{i-1})} e^2 \tan B_{i-1}, \quad H_i = D/\cos B_{i-1} = Z/\sin B_{i-1} - N(1 - e^2)$$

B. R. Bowring described a closed formula[1] that is more effective and sufficiantly accurate and that is used in GNU Gama.



The centre of curvature C of the spheroid corresponding to P' is the point

$$(e^2 a \cos^2 u, -e'^2 b \sin^3 u),$$

where $e'^2 = (a^2 - b^2)/b^2$ is second eccentricity and u is the parametric latitude of the point P', $(1 - e^2)N \sin B = b \sin u$. Therefore

$$\tan B = \frac{Z + e'^2 b \sin^3 u}{D - e^2 a \cos^3 u}.$$

This is clearly an iterative solution; but it has been found that this formula is extremely accurate using the single first approximation for u for the $\tan u = (Z/D)(a/b)$. Maximum error in earth bound region is 3e-8 of sexagesimal arc seconds (5e-7 millimetres); maximum is 0.0018" (0.1 millimetres) at height H = 2a.

6.4 Class `g3::Model`

g3::model documentation shall come here ...

```
namespace GNU_gama {  namespace g3 {


  class Model {
```

---

[1] B. R. Bowring: Transformation from spatial to geographical coordinates, Survey Review XXIII, 181, July 1976

```
  public:

    typedef GNU_gama::PointBase<g3::Point>              PointBase;
    typedef GNU_gama::ObservationData<g3::Observation>  ObservationData;

    PointBase           *points;
    ObservationData     *obs;

    GNU_gama::Ellipsoid  ellipsoid;


    Model();
    ~Model();

    Point* get_point(const Point::Name&);
    void   write_xml(std::ostream& out) const;
    void   pre_linearization();
}}
```

7 Gama-local test suite

GNU Gama comes with a set of tests that provides `gama-local` test suite. To run the test suite, go to the top-level Gama directory and type

```
$ make check
```

You should see the names of the test suite files as they are processed, any other output indicates some problem. The output might be for example example this

```
Entering directory '/home/cepek//gama/tests/gama-local'
PASS: gama-local-version
PASS: gama-local-adjustment
PASS: gama-local-algorithms
PASS: gama-local-xml-xml
PASS: gama-local-html
PASS: gama-local-equivalents
PASS: gama-local-xml-results
PASS: gama-local-parameters
PASS: gama-local-updated-xml
PASS: gama-local-sqlite-reader
PASS: xmllint-gama-local-xsd
PASS: xmllint-gama-local-adjustment-xsd
============================================================================
Testsuite summary for gama 2.14
============================================================================
# TOTAL: 12
# PASS:  12
# SKIP:  0
# XFAIL: 0
# FAIL:  0
# XPASS: 0
# ERROR: 0
============================================================================
```

Number of tests vary according to the configuration of your system. Tests that are always present are

```
gama-local-version
gama-local-adjustment
gama-local-algorithms
gama-local-xml-xml
gama-local-html
gama-local-equivalents
gama-local-xml-results
gama-local-parameters
gama-local-updated-xml
```

Optional tests are

```
gama-local-sqlite-reader
xmllint-gama-local-xsd
```

```
xmllint-gama-local-adjustment-xsd
```

which are included only if sqlite3 database support libraries and/or `xmllint` program are installed.

7.1 Internal organisation

Gama-local tests are implemented as shell scripts that are stored in `gama/tests/gama-local` directory. The scripts are generated from corresponding `.in` files which are stored in `gama/tests/gama-local/script` directory where are also stored helper C++ programs called by the testing suite scripts. Generating scripts and the build of helper programs is controlled from `gama/tests/gama-local/Makefile.am`, where a list of testing data files is also defined.

In `gama/tests/gama-local` directory are also stored detail `.log` files for all tests together with corresponging `.trs` (as in **T**est **R**e**S**ults) files.

All files generated by the test suite are stored in `gama/tests/gama-local/script/2.14` (thus generated files from different versions are not overwritten).

To run selected test individually, go to the directory `gama/tests/gama-local` and start the test manually

```
$ cd gama/tests/gama-local
$ ./test-name
```

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Version 1.1, March 2000

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